A Stock-Flow Model of Keynes' Theory of Interest

Chapter 5 in
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Abstract

A Stock-Flow Model of Keynes' Theory of Interest provides a formal model of Keynes' Liquidity-Preference theory of interest that draws a clear distinction between stocks and flows. This model is used to clarify the confusion that exists within the discipline of economics with regard to the issues that separated Robertson and Keynes Loanable - funds / Liquidity - Preference controversy. Robertson's misconceptions with regard to the nature of Keynes' theory of interest, and by extension those of Harry Johnson, Axel Leijonhufvud, George Horwich, Meir Kohn, Sho-Chieh Tsiang and others who have defended Robertson's arguments, are explained within the context of this model.

Chapter 5:
A Stock-flow Model of Keynes' Theory of Interest

In specifying a model of Keynes' liquidity-preference theory of interest it is necessary to abandon the models specified by Robertson and his followers such as Sho-Chieh Tsiang and Meir Kohn since their models do not provide a meaningful distinction between the demand for money that arises as a result of "an investment decision" and "the demand for active balances which will arise as a result of the investment activity itself." (Keynes, 1937a, pp. 246-8; Blackford, 2020a, pp. 26-77; 2020b, ch. 4; Bibow 1995) Nor do they draw a clear distinction between stocks and flows. Thus, in order to explain Keynes' theory it is necessary to extend and reformulate the Robertsonian period model in such a way as to incorporate Keynes' concept of the demand for 'finance' and to avoid the conflation of stocks and flows one finds in the typical Robertsonian model.

I. A Model of Keynes' Liquidity Preference Theory

A three-sector model consisting of firms, households, and banks is specified in this section.

a. Firms

Demand for Money by Firms

Keynes' concept of the demand for finance presupposes that at least a portion of the money required to finance expenditures cannot be obtained from the ordinary cash flows of business (e.g., expenditures on real estate, heavy equipment, *increases* in the *flow* of expenditures that are normally financed out of the ordinary cash flows of business, etc.) and must be se-

cured otherwise *before* the expenditure can take place. This assumption is incorporated in the model by assuming there is a demand for money on the part of firms *at each point in time* (t) associated with such expenditures (F_t^f) that is a function (f^f) of the current rate of interest (R_t):

(1) $F_t^f = f^f(R_t), \qquad f^{f'} < 0,$

where f^f represents the 'finance' demand function of firms, and F_t^f is the *amount* of money firms demand at time t for the purpose of financing real and financial expenditures that cannot be financed out of the ordinary cash flow of business.[1] (See Bibow, 1995; Blackford, 2020b, ch.4; and the discussion of Bibow in footnote 11 below.)

Firms also demand money for the purpose of financing ongoing real and financial investment expenditures (e.g., tools, supplies, maintenance and repair, inventories, repayment of debt, etc.) that are routinely financed through the ordinary cash flow of business. The demand for these balances (F_t^i) is assumed to depend on the *rate* at which income is produced (Y_t) and is given by:

(2) $F_t^i = f^i(Y_t), \qquad f^{i'} > 0.$

Thus, F_t^f and F_t^i each represent separate sources of demand for money on the part of firms in that firms hold an amount of money F_t^i that is demanded for the purpose of financing expenditures that can be financed out of ordinary cash flows, and, at the same

[1] Each of the equations specified in this model is defined in terms of a single independent variable. Other independent variables that determine the dependent varible are assumed to be subsumed in the functional form of the equation and determine the position of the bivariate graph of the function.

time, firms hold an *amount* of money F_t^f that is demanded in anticipation of expenditures that cannot be financed out of ordinary cash flows.

Firms also make income payments (e.g., wages, dividends, rent, interest, etc.) to households that must be financed. The amount of money demanded for this purpose (F_t^y) is assumed to depend on income Y_t as determined by *the value of output produced* (Keynes, 1936, pp. 52-63; Hayes; Blackford, 2020a, pp. 26-77; 2020b, chs. 2, 3):

(3) $F_t^y = f^y(Y_t), \qquad f^{y'} > 0.$

Firms demand money for precautionary and speculative purposes as well where the quantity of money demanded for precautionary purposes (F_t^p) is assumed to depend on income Y_t:

(4) $F_t^p = f^p(Y_t), \qquad f^{p'} > 0,$

and the quantity of money demanded for speculative purposes (F_t^s) is assumed to depend on the rate of interest R_t:

(5) $F_t^s = f^s(R_t), \qquad f^{s'} < 0.$

Thus, F_t^f, F_t^i, F_t^y, F_t^p, and F_t^s each represent separate sources of demand for money by firms, and the aggregate demand by firms (F_t^{md}) is given by the sum of these individual sources of demand:

(6) $$\begin{aligned} F_t^{md} &= F_t^f + F_t^i + F_t^y + F_t^p + F_t^s \\ &= f^f(R_t) + f^i(Y_t) + f^y(Y_t) + f^p(Y_t) + f^s(R_t). \end{aligned}$$

Money Available to Firms

There are two sources of money available to firms to satisfy their demands for money during the period. Firms begin each period with a certain amount of money carried forward from the previous period. The amount of these balances is given by F_{t-x} where x identifies the length of the period being examined, be-

ginning at time $\boldsymbol{t - x}$ and ending at time $\boldsymbol{t}$. In addition, firms take in money during the period through the sale of consumption goods, investment goods, and non-debt assets. Firms also pay out money in income payments and as they purchase investment goods and non-debt assets. Thus, the amount of money held by firms at the end of the period ($\boldsymbol{F_t^n}$) is determined by a) the amount of money held by firms at the beginning of the period $\boldsymbol{F_{t-x}}$ plus b) the *rates* at which money is received by firms from the sale of investment ($\boldsymbol{I_t}$) and consumption ($\boldsymbol{C_t}$) goods and the *rate* at which money is transferred to firms from households ($\boldsymbol{T_t^n}$) as a result of the *net* sale of non-debt assets to households during the period less the *rates* at which money is paid out by firms in income payments ($\boldsymbol{Y_t^p}$) and the purchase of investment goods ($\boldsymbol{I_t}$) during the period all integrated over the interval of time $\boldsymbol{x}$ being examined:[2]

$$(7)\quad F_t^n = F_{t-x} + \int_{t-x}^{t}\left((C_t + I_t + T_t^n) - (Y_t^p + I_t)\right)dt$$
$$= F_{t-x} + \int_{t-x}^{t}\left(C_t + T_t^n - Y_t^p\right)dt.$$

Demand for Credit by Firms

A third source of money available to firms is credit. It is assumed that borrowing, lending, and the repayment of debt occur only at the end of the period[3]

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[2] The integral notation is used only to indicate the need to sum expenditures and receipts over the period and can be replaced with that of summation if one wishes to do so.

[3] The assumption that borrowing, lending, and the repayment of debt do not take place during the period is made to simplify the exposition only. In the absence of this assumption terms would have to be added to (**7**), (**14**), and (**17**) to account for the transfer of money that result from these transactions during the period. These terms would also appear in (**8**) and (**19**) which are cluttered enough as it is. These terms would then cancel and not appear in the equilibrium condition (**22**) and (**23**). As a result, the assumption that borrowing, lending, and the repayment

and that firms borrow only to meet their financial needs for *money* and lend only to dispose of excess *money* balances they have no use for otherwise or in the case of trade credit to facilitate current transactions that provide for the payment of *money* at a later date. It is also assumed that, in general, firms are net demanders of loanable funds where the *net* contribution of firms to the *quantity* of loanable funds demanded at the end of the period is given by the *amount* of money demanded by firms at the end of the period F_t^{md} (6) less the *amount* of money held by firms at the end of the period F_t^n (7). Thus, the aggregate demand for loanable funds by firms at the end of the period (L_t^d) is given by:[4]

$$(8)\quad L_t^d = \left(F_t^f + F_t^i + F_t^y + F_t^p + F_t^s\right) - F_t^n$$
$$= \left(f^f(R_t) + f^i(Y_t) + f^y(Y_t) + f^p(Y_t) + f^s(R_t)\right)$$
$$- \left(F_{t-x} + \int_{t-x}^{t}\left(C_t + T_t^n - Y_t^p\right)dt\right).$$

b. Households

Demand for Money by Households

Households are also assumed to hold money for precautionary and speculative purposes where the amount of money demanded for precautionary purposes (H_t^p) is assumed to depend on income Y_t:

$$(9)\quad H_t^p = h^p(Y_t), \qquad h^{p'} > 0,$$

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of debt do not take place during the period has no effect on the conclusions that follow from the equilibrium conditions of the model.

[4] Even though firms are generally considered to be net demanders of loanable funds (L_t^d positive), it is possible for firms to be net suppliers of loanable fund (L_t^d negative). This will occur whenever the financial needs of firms allow firms to lend and repay debt in excess of their borrowing at the end of the period. It should also be noted that L_t^d is the net *amount* of money firms wish to borrow at time t; L_t^d is *not* the net *rate* at which firms wish to borrow money.

and the amount demanded for speculative purposes (H_t^s) is assumed to depend on the rate of interest R_t:

(10) $H_t^s = h^s(R_t), \qquad h^{s'} < 0.$

Households also demand money to finance ongoing consumption and other real and financial expenditures that are normally financed through the flow of income (e.g., food, clothing, utilities, repayment of debt, etc.), the amount of which (H_t^c) is assumed to depend on income Y_t:

(11) $H_t^c = h^c(Y_t), \qquad h^{c'} > 0.$

At the same time households demand money in anticipation of expenditures that cannot be financed through the flow of income (e.g., vacations, durable goods, *increases* in the *flow* of expenditures that cannot be financed out of income, etc.). The amount of money demanded for this purpose (H_t^f) is assumed to depend on the rate of interest R_t:

(12) $H_t^f = h^f(R_t), \qquad h^{f'} < 0.$

In this specification, h^f represents households' 'finance' demand function and H_t^f the amount of money households demand to finance expenditures that cannot be financed out of income as opposed to balances demanded to finance ongoing expenditures that are normally financed out of income H_t^c. Thus, H_t^p, H_t^s, H_t^c, and H_t^f each represent separate sources of demand for money by households, and the aggregate demand by households (H_t^{md}) is given by the sum of these individual sources of demand:

(13) $$H_t^{md} = H_t^p + H_t^s + H_t^c + H_t^f$$
$$= h^p(Y_t) + h^s(R_t) + h^c(Y_t) + h^f(R_t).$$

Money Available to Households

There are two sources of money available to households to satisfy their demands for money during

the period. Households, as with firms, begin each period with a quantity of money carried forward from the previous period (H_{t-x}) which is the amount of money held by households at the beginning of the period, that is, at time ***t-x***. In addition, households obtain money during the period by way of income payments from firms and pay out money as they purchase consumption goods and non-debt assets. Thus, the amount of money held by households at the end of the period (H_t^n) is determined by a) the amount of money held by households at the beginning of the period H_{t-x} plus b) the *rate* at which money is received in income payments from firms during the period Y_t^p less the *rates* at which households spend on consumption goods C_t and the rate at which money is transferred from households to firms through the *net* purchase of non-debt assets T_t^n during the period integrated over the interval of time ***x*** being examined:

$$(14) \quad H_t^n = H_{t-x} + \int_{t-x}^{t} (Y_t^p - C_t - T_t^n) dt.$$

Supply of Credit by Households

Households also have access to the credit market at the end of the period. With regard to credit, as with firms, it is assumed that households borrow only to meet their financial needs for *money* and lend only to dispose of excess *money* balances they have no use for otherwise. It is also assumed that, in general, households are net suppliers of loanable funds and that the *net* contribution of households to the *quantity* of loanable funds supplied (L_t^s) is given by the net *amount* of money held by households H_t^n (**14**) at the end of the period less the *amount* of money demanded by households H_t^{md} (**13**) at the end of the period:

$$(15)\quad L_t^{hs} = H_t^n - (H_t^p + H_t^s + H_t^c + H_t^f)$$
$$= \left(H_{t-x} + \int_{t-x}^{t}(Y_t^p - C_t - T_t^n)dt\right) -$$
$$\left(h^p(Y_t) + h^s(R_t) + h^c(Y_t) + h^f(R_t)\right).$$

c. Banks

Money is created by banks through the process of making loans where the stock of money created by banks (M_t^s) depends on the rate of interest:[5]

$$(16)\quad M_t^s = m^s(R_t), \qquad m^{s'} > 0.$$

Banks contribute to the supply of loanable funds (L_t^{bs}) by the difference between the supply of money at the end of the period M_t^s and the amount that exists at the beginning of the period (M_{t-x}):

$$(17)\quad L_t^{bs} = M_t^s - M_{t-x}$$
$$= m^s(R_t) - M_{t-x},$$

where M_{t-x} is the quantity of money held by firms and households at the beginning of the period:

$$(18)\quad M_{t-x} = F_{t-x} + H_{t-x}.$$

d. Equilibrium Conditions

The aggregate demand for loanable funds (L_t^d) is assumed to be given by the *net* demand for loanable funds of firms (**8**), and the aggregate supply of loanable funds (L_t^s) is given by the sum of the net supply

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[5] Keynes generally assumed the supply of money to be exogenously determined by the monetary authorities in the *General Theory*, but in December of 1937 he noted that:

> Dr. Herbert Bab has suggested to me that one could regard the rate of interest as being determined by the interplay of the terms on which the public desires to become more or less liquid and those on which the banking system is ready to become more or less unliquid. This is, I think, an illuminating way of expressing the liquidity-theory of the rate of interest.... (1937b, p.666),

Thus, the supply of money is considered to be endogenous in the text above, but it can be assumed to be exogenous if one wishes. Cf., Bibow (2000b).

provided by banks $\boldsymbol{L_t^{bs}}$ (**17**) and households $\boldsymbol{L_t^{hs}}$ (**15**):

$$(19)\quad L_t^s = (M_t^s - M_0) + \left(H_t^n - \left(H_t^p + H_t^s + H_t^c + H_t^f\right)\right)$$
$$= \left(m^s(R_t) - M_{t-x}\right) + \left(H_{t-x} + \int_{t-x}^{t}\left(Y_t^p - C_t - T_t^n\right)dt\right) -$$
$$\left(h^p(Y_t) + h^s(R_t) + h^c(Y_t) + h^f(R_t)\right).$$

It is also assumed that the aggregate demand for money is given by the sum of the money demanded by firms $\boldsymbol{F_t^{md}}$ (**6**) and households $\boldsymbol{H_t^{md}}$ (**13**):

$$(20)\quad M_t^d = \left(F_t^p + H_t^p + F_t^s + H_t^s\right)$$
$$+ \left(\left(F_t^f + H_t^f\right) + \left(H_t^c + F_t^i + F_t^y\right)\right)$$
$$= \left(f^p(Y_t) + h^p(Y_t) + f^s(R_t) + h^s(R_t)\right) +$$
$$\left(\left(f^f(R_t) + h^f(R_t)\right) + \left(h^c(Y_t) + h^c(Y_t) + f^i(Y_t) + f^y(R_t)\right)\right).$$

Given the way in which the supplies and demands for loanable funds are defined in terms of the demands for money, the demand for loanable funds (**8**) will equal the supply of loanable funds (**19**) if, and only if, the supply of money (**16**) is equal to the demand for money (**20**). This can be seen by setting $\boldsymbol{L_t^d}$ from (**8**) equal to $\boldsymbol{L_t^s}$ from (**19**) to obtain the equilibrium condition for the loanable funds market:

$$(21)\quad \left(F_t^f + F_t^i + F_t^y + F_t^p + F_t^s\right) - \left(F_{t-x} + F_t^n\right)$$
$$= \left(M_t^S - M_{t-x}\right)$$
$$+ \left(\left(H_{t-x} + H_t^n\right) - \left(H_t^p + H_t^s + H_t^c + H_t^f\right)\right)$$
$$\left(f^f(R_t) + f^i(Y_t) + f^y(Y_t) + f^p(Y_t) + f^s(R_t)\right)$$
$$- \left(F_{t-x} + \int_{t-x}^{t}\left(C_t + T_t^n - Y_t^p\right)dt\right)$$
$$= \left(\left(m^s(R_t) - M_{t-x}\right) + \left(H_{t-x} + \int_{t-x}^{t}\left(Y_t^p - C_t -\right.\right.\right.$$
$$\left.\left.\left.T_t^n\right)dt\right)\right) - \left(h^p(Y_t) + h^s(R_t) + h^c(Y_t) + h^f(R_t)\right),$$

Since a) the amount of money held by firms $\boldsymbol{F_{t-x}}$ and households $\boldsymbol{H_{t-x}}$ at the beginning of the period sum to $\boldsymbol{M_{t-x}}$ (**18**), and b) the amount of money *received* by

households from firms during the period that is not returned to firms by way of the consumption goods and non-debt asset markets during the period is equal to $\int_{t-x}^{t}\left(\boldsymbol{Y_t^p - C_t - T_t^n}\right)\boldsymbol{dt}$ and appears on both sides of (**21**) these terms cancel and (**21**) reduces to:

$$(22)\ \left(F_t^f + F_t^i + F_t^y + F_t^p + F_t^s\right) = M_t^S - \left(H_t^p + H_t^s + H_t^c + H_t^f\right),$$

$$\left(f^f(R_t) + f^i(Y_t) + f^Y(Y_t) + f^p(Y_t) + f^s(R_t)\right) = m^s(R_t) - \left(h^p(Y_t) + h^s(R_t) + h^c(Y_t) + h^f(R_t)\right).$$

And by rearranging the terms in (**22**) we obtain the equilibrium condition for the supply and demand for money:

$$(23)\quad M_t^S = \left(F_t^p + H_t^p + F_t^s + H_t^s\right) + \left(\left(F_t^f + H_t^f\right) + \left(H_t^c + F_t^i + F_t^y\right)\right)$$

$$m^s(R_t) = \left(f^p(Y_t) + h^p(Y_t) + f^s(R_t) + h^s(R_t)\right) + \left(\left(f^f(R_t) + h^f(R_t)\right) + \left(h^c(Y_t) + h^c(Y_t) + f^i(Y_t) + f^y(Y_t)\right)\right).$$

Thus, equilibrium in the loanable funds market (**21**) implies equilibrium in the supply and demand for money (**23**) and vice versa. As has been noted, this follows directly from the way in which the contributions of firms, households, and banks to the supply and demand for loanable funds are specified in terms of the sources of *money* available to decision-making units during the period and the amounts of *money* demanded on the part of decision-making units at the end of the period; for the credit market to clear, the aggregate demand for and supply of the *stock* of money must be equal.

What is of particular interest here, however, is that the form of the money demand function given by (**20**) and the right-hand side of (**23**) corresponds to the form of the money demand function described by Keynes in his final attempt to explain his theory of in-

terest to Robertson:

> The rate of interest is determined by the total demand and total supply of cash or liquid resources. The total demand falls in two parts: the inactive demand due to the state of confidence and expectation on the part of the owners of wealth, and the active demand due to the level of activity established by the decisions of the entrepreneurs. The active demand in its turn falls into two parts: the demand due to the time-lag between the inception and the execution of the entrepreneur's decisions, and the part due to the time-lags between the receipt and the disposal of income by the public and also between the receipt by entrepreneurs of their sale-proceeds and the payment by them of wages, etc. (1938, p. 319)

The money demand function (**20**) is organized such that $(\boldsymbol{F_t^p + H_t^p + F_t^s + H_t^s})$ is "the inactive demand for money due to the state of confidence and expectation on the part of the owners of wealth;" $(\boldsymbol{F_t^f + H_t^f})$+$(\boldsymbol{H_t^c + F_t^i + F_t^y})$ is "the active demand due to the level of activity established by the decisions of the entrepreneurs" which "falls into two parts:" $(\boldsymbol{F_t^f + H_t^f})$ which is "the demand due to the time-lag between the inception and the execution of the entrepreneur's [and household's] decisions" and $\boldsymbol{(H_t^c + F_t^i + F_t^y)}$ is "the part due to the time-lags between the receipt and the disposal of income by the public and ... the receipt by entrepreneurs of their sale-proceeds and the payment by them of wages, etc."

The correspondence between the right-hand-side of (**20**) and Keynes' summary of the demand for money makes it possible to use the model of which (**20**) is a part to sort through the "verbal tangles" (Robertson, 1940, p. 9) and discover the nature of the fundamental differences between Robertson's and

Keynes' theories of interest. This can be accomplished by examining the supplies and demands for money and loanable funds specified above, and it is instructive to include a third set of functions, namely, the supply and demand for speculative balances where the demand for speculative balances ($\boldsymbol{Q_t^d}$) is *defined* by the sum of the speculative demands for money by households (**10**) and firms (**5**):

$$(24)\quad \begin{aligned} Q_t^d &= F_t^s + H_t^s \\ &= f^s(R_t) + h^s(R_t), \end{aligned}$$

and the supply of speculative balances ($\boldsymbol{Q_t^s}$) is *defined* by the supply of money (**16**) less the demands for transactions and precautionary purposes (**1**) - (**4**), (**9**), (**11**) and (**12**), that is, less all of those demands for money that firms and households have a use for other than to lend or hold as an asset (cf., Keynes, 1936, p. 171):

$$(25)\quad \begin{aligned} Q_t^s &= M_t^s - (F_t^i + F_t^f + H_t^c + H_t^f + F_t^y + F_t^p + H_t^p) \\ &= m^s(R_t) - (f^i(Y_t) + f^f(R_t) + h^c(Y_t) + h^f(R_t) \\ &\qquad + f^y(Y_t) + f^p(Y_t) + h^p(Y_t)). \end{aligned}$$

The significance of (**24**) and (**25**) is that, given the level of income $\boldsymbol{Y_t}$, the demand for speculative balances (**24**) identifies the amount of money demanded $\boldsymbol{Q_t^d}$ to satisfy the speculative motive at each rate of interest, and the supply of speculative balances (**25**) identifies the amount of money $\boldsymbol{Q_t^s}$ that is not demanded for transactions and precautionary purposes and, therefore, is the amount of money that is available to satisfy the demand for speculative balances at each rate of interest. (Keynes, 1936, chs. 13, 15, 17)

Finally, it must be noted that the supply (**19**) and demand (**8**) for loanable funds are *not* independent behavioral equations in this model. Since the supply and demand for loanable funds are explicitly defined

in terms of the supply and demand for money they are *ex post* magnitudes, determined by the supply and demand for money as dictated by the desired transactions of decision-making units as determined by the implicit behavioral equations of the model. (Blackford 2020a, pp. 26-77; 2020b, ch. 2-4)

II. Thriftiness, Investment, and Finance

In examining the issues that separated Robertson and Keynes we begin with the effects of a *ceteris paribus* increase in thriftiness, that is, an increase in the propensity to save, in a situation in which income and the supply and demand for money are unchanged.

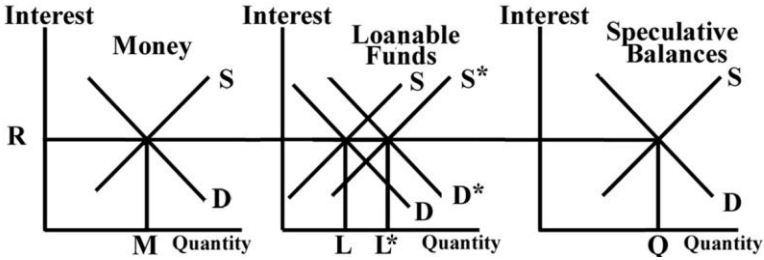
a. An Increase in Thriftiness

A *ceteris paribus* increase in thriftiness, given income $\boldsymbol{Y_t}$ and the supply (**16**) and demand (**20**) for money, means a decrease in the rate of consumption expenditures $\boldsymbol{C_t}$. The reduced flow of consumption expenditures $\boldsymbol{C_t}$ from households to firms must decrease the *flow* of money made available to firms through the consumption-goods market while the supplies and demands for money and speculative balances remain unchanged since the flow of consumption expenditures $\boldsymbol{C_t}$ does not enter the functions (**16**) (**20**) (**24**) (**25**) that determine these supplies and demands. Given the level of income $\boldsymbol{Y_t}$ firms must turn to the credit (or asset) market(s) to maintain their transactions and precautionary balances. At the same time, households must turn to the credit (or asset) market(s) to find an outlet for the excess money balances accumulated through the process of saving that exceed the balances they wish to hold.[6] This situation

[6] The way in which the prices of assets are determined is explained in Blackford (2020b, ch. 6).

is illustrated in **Figure 1** where the initial supplies and demands for money, loanable funds, and speculative balances (**16**), (**20**), (**19**), (**8**), (**24**), and (**25**) are given by their respective supply **S** and demand **D** curves, and **R**, **M**, **L**, and **Q** represent the initial equilibrium values for the rate of interest, money, loanable funds, and speculative balances, respectively.

Figure 1: An Increase in Thriftiness.[7]



The effects of an increase in thriftiness are illustrated in this figure by an increase in the supply of loanable funds (**19**) from **S** to **S*** that is accompanied by an increase in the demand for loanable funds (**8**) from **D** to **D***. Since C_t does not enter the functions that determine the positions of the supplies **S** and demands **D** for money (**16**) (**20**) and speculative balances (**24**) (**25**) there is no reason for these curves to change. As a result, the amount of money obtained by firms through the credit market increases from **L** to **L***, but since the supply **S** and demand **D** for money remain unchanged the fall in C_t must cause the loana-

[7] It should be noted that each of these three sets of curves must intersect at the same rate of interest since, by virtue of the way in which these functions are defined in terms of the supply and demand for money, the markets for credit, money, and speculative balances describe the same market, namely, what Irving Fisher referred to as "the money market" (1930, p. 12). It is simply assumed that the aggregate supply and demand for loanable funds are positive in this figure.

ble-funds supply **S** and demand **D** curves to increase by exactly the same amount ($\partial L_t^s/\partial C_t = \partial L_t^d/\partial C_t < 0$), and the resulting loanable-funds **S*** and **D*** curves must intersect at the same equilibrium rate of interest as the initial loanable-funds supply **S** and demand **D** curves. Thus, there is no reason for **R** to change nor is there any reason for **M** or **Q** to change as a *direct* result of the increase in the propensity to save.[8]

Thus, given the supply and demand for money a *ceteris paribus* increase in thriftiness that leaves income and the supply and demand for money unchanged cannot have a *direct* effect on either the rate of interest **R**, the size of the stock of money **M**, or the amount of money held for speculative purposes **Q** in this model, but it does increase the demand **D** (**8**) and supply **S** (**19**) of loanable funds and, therefore, the amount of money borrowed and lent **L***. At the same time, it is important to recognized that there is no *a*

[8] Note that the flow of money payments from firms to households Y_t is 'released' by households and returned to firms in this model whether it is released by way of consumption expenditures C_t during the period or by way of savings $Y_t - C_t$ through lending L_t^{hs} or the purchase of non-debt assets T_t^n and added to the supply of loanable funds at the end of the period. Gven income Y_t and the supply and demand for money, whether Y_t is spent on consumption goods C_t or saved $Y_t - C_t$ and lent L_t^{hs} or used to purchase non-debt assets T_t^n has no effect on the equilibrium conditions (**22**) and (**23**). In either case, the aggregate amount of money made available to firms is the same. It was Robertson's (1938) inability to understand how money is released in this way that prompted Keynes to declare: "It is Mr. Robertson's incorrigible confusion between the revolving fund of money in circulation and the flow of new saving which causes all his difficulties." (1938, p. 322) The *flows* of consumption C_t and saving $Y_t - C_t$ are sustained by the revolving *stock* of money in circulation, but the *flow* of saving $Y_t - C_t$ is not the same thing as the *stock* of money M_t, and it is money M_t that firms need to finance their transactions, not savings $Y_t - C_t$. See Blackford (2020b, ch. 4), and cf., Tsiang and Kohn.

priori reason to believe the demand and supply of money will remain unchanged in response to an increase in the propensity to save in the real world [9] or that the changes in the individual components of the demand for money that may occur will necessarily offset each other in any given situation.[10]

The resulting fall in sales in the consumption goods industries that results from a decrease in the propensity to consume, for example, is apt to be met at least initially and partially by a decrease in the demand for precautionary $\boldsymbol{F_t^p}$ and speculative $\boldsymbol{F_t^s}$ balances by firms. By the same token, the assumption that

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[9] Since firms have a choice between borrowing money or selling assets to obtain the needed funds in this situation, and households have a choice between lending money and buying assets in order to dispose of their excess balances to the extent the choices of households and firms are not compatible at the existing rate of interest and price of non-debt assets the rate of interest and the price of non-debt assets can be expected to change to make them compatible. It is important to note, however, that these are protfolio balance decissions that involve changes in the supplies and demands for money and non-debt assets, not saving and investment decisions. Since a) there is no *a priori* reason to believe that changes in saving or investment determine the nature of the incompatibility, b) the change in the rate of interest and price of assets can go either way depending on the nature of the incompatibility, and c) these kinds of changes in the rate of interest and price of assets can and do occur even when there is no change in saving or investment they must be explained in terms of changes in the supplies and demands for money and non-debt assets; they cannot be explained in terms of changes in saving or investment. See Keynes (1930, pp. 130-1) and Blackford (2020b, ch. 1).

[10] The assumption that the demand for money remains unchanged in this *ceteris paribus* situation is equivalent to the assumption that the right hand side of:

$$\begin{aligned}\boldsymbol{\partial\left(M_t^d\right)/\partial C_t} = {} & \boldsymbol{\partial(f^p(Y_t) + h^p(Y_t) + f^s(R_t) + h^s(R_t))/\partial C_t} \\ & + \boldsymbol{\partial((f^f(R_t) + h^f(R_t)) + (f^i(Y_t) + h^c(Y_t)} \\ & \boldsymbol{+ f^y(Y_t)))/\partial C_t}\end{aligned}$$

sums to zero. There is no *a priori* reason to believe this must hold at all points in time in any given situation in the real world.

households' demand for money does not change in the *ceteris paribus* situation represented in **Figure 1** presupposes either that the demand for consumption balances H_t^c does not change or that any change in H_t^c that arises from the decrease in C_t is offset by a corresponding change in finance H_t^f, precautionary H_t^p, or speculative H_t^s balances. (Blackford, 2020b, ch. 4) To the extent this is not the case the supply **S** and demand **D** for money cannot remain unchanged the change in the supply of loanable funds **S** cannot be equal to the change in the demand for loanable funds **D** in **Figure 1**, and the equilibrium rate of interest **R** must change.

What actually happens to the supplies and demands for money, loanable funds, and speculative balances as the system evolves *through time* will depend on the extent to which these kinds of changes in the money demand functions of households and firms occur. This is simply a logical/mathematical fact, but the significance of Keynes' liquidity preference theory of interest is not that it ignores these kinds of changes and effects, but that—*unlike the loanable-funds theory*—Keynes' liquidity preference theory provides a *logically consistent analytical framework* within which it is possible *to understand, explain, and predict* how and why these kinds of changes occur and *to understand, explain, and predict* how and why they will affect the rate of interest in the real world. What makes this possible is Keynes' realization that it is the choices of households and firms with regard to their demands for money (i.e. *liquidity*) and of banks with regard to the supply of money that determine the rate of interest *at each point in time*, not their choices with regard to saving and investment, irrespective of the

independent variables that may appear in the supply or demand for money functions or the changes that may occur in these functions. (See Bibow, 2000a; 2001; Blackford, 2020a; 2020b, and cf., Leijonhufvud; Horwich; Tsiang; and Kohn.)

b. The Demand for Finance

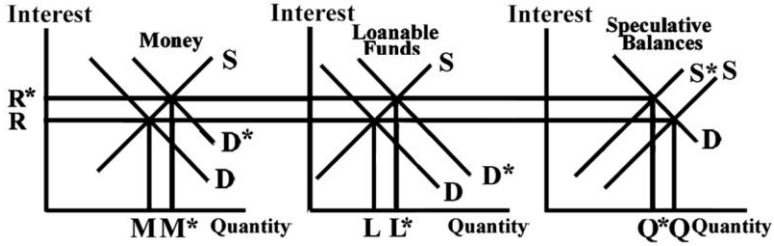
Jörg Bibow (1995) has observed that an *increase* in the rate of either investment $\boldsymbol{I_t}$ or consumption $\boldsymbol{C_t}$ expenditures will necessitate an increase in the demand for finance on the part of firms $\boldsymbol{f^f}$ or households $\boldsymbol{h^f}$ *before* the increase in investment $\boldsymbol{I_t}$ or consumption $\boldsymbol{C_t}$ expenditures can actually occur. This phenomenon is easily illustrated in the model specified above.

A *ceteris paribus* increase in the MEC, for example, must increase the finance demand of firms $\boldsymbol{F_t^f}$ $(= \boldsymbol{f^f(R_t)})$, and *there is no reason to believe any other demand for money in the system will decrease* since the motivation for holding other balances has not changed. Given the supply of money and the other demands for money in the system, the implied increase in expenditures on investment goods $\boldsymbol{I_t}$ can only be financed if new balances are created or the quantities of other demands for money are reduced. To bring this about firms must bid up the rate of interest to induce households and firms to reduce the quantity of speculative and finance balances demanded or banks to increase the quantity of money in existence *before* an increase in expenditures on investment goods can occur. This situation is illustrated in **Figure 2** where **S**, **D**, **R**, **L**, **M**, and **Q** are the initial curves and values as defined in **Figure 1**.

The increase in the firm sector's demand for fi-

nance f^f (1) must increase the aggregate demands for

Figure 2: An Increase in Demand for Finance.



money (**20**) and loanable funds (**8**) as indicated by the shifts in these curves from **D** to **D*** in this figure. At the same time, the supply of speculative balances (**25**) must fall as is indicated by the shift in the speculative balance supply curve from **S** to **S***. The result is a decrease in speculative balances and increase in the rate of interest from **Q** and **R** to **Q*** and **R*** as the amount of money borrowed/lent and the stock of money increase from **L** and **M** to **L*** and **M***, respectively.[11]

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[11] Cf., Keynes:

> An increase in activity raises the demand for cash, first of all to provide for the first of these time-lags [the time-lag between the inception and execution of the entrepreneurs' decisions] in circulation, and then to provide for the second of them [the time-lag between the receipt and disposal of income … and … sale-proceeds]. Thereafter the demand for cash falls away unless the completed activity is being succeeded by a new activity. A given stock of cash provides a revolving fund for a steady flow of activity; but an increased rate of flow needs an increased stock to keep the channels filled. When decisions are made which will lead to an increase in activity, the effect is first felt in the demand for more cash for " finance." (1938, p. 319)

Bibow (1995) has demonstrated that in the case of expenditures that are normally financed through money received from current cash flows (tools, etc.) the increase in f_t^f or h_t^f will be temporary, lasting only

c. Planned Activity and Congestion

Robertson argued that the increase in "congestion in the capital market" that results from the increase in planned investment expenditures seen in **Figure 2** can be relieved through an increase in saving (Robertson, 1938, p.318; and cf. Blackford, 2020b, chs. 2-4; Tsiang; Horwich; and Kohn). It should be obvious from the above why this is not the case. An increase in $\boldsymbol{f^f}$ causes an increase in the demand for *money* on the part of firms to finance their increased planned investment expenditures, and, as we saw in the analysis of **Figure** 1, an increase in saving cannot *in itself* provide the additional finance, that is, *money*, needed by firms to finance these expenditures. It can provide an increase in the *flow* of *savings*, and, thereby, the *flow of money* made available to firms through the loanable funds market, but *it also reduces the flow of money made available to firms through the consumption goods market by the same amount*. The additional 'finance' needed by firms consists of *money,* not savings. Since there is no change in the *net flow of money* made available to firms through the combined goods and loanable-funds markets as a result of an increase in saving, an increase in saving cannot, *in itself*, increase the amount of 'finance', that is, *money*, made available to firms and, therefore,

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until income changes sufficiently to allow the increased flow of expenditures to be financed in the way they are normally financed through an increase in the transactions balances of firms $\boldsymbol{F_t^i}$ $(= \boldsymbol{f^i(Y_t)})$ or households $\boldsymbol{H_t^c}$ $(= \boldsymbol{h^c(Y_t)})$. It is only a permanent increase in those kinds of expenditures that are *not* normally financed through sales and income (e.g., real estate, vacations, etc.) that a permanent increase in the level of expenditures will lead to a permanent increase in $\boldsymbol{f^f}$ or $\boldsymbol{h^f}$. In either case "the demand for cash falls away unless the completed activity is being succeeded by a new activity."

cannot *in itself* relieve the congestion in the capital market in this *ceteris paribus* situation.[12]

d. Thriftiness and Income

It is clear from the equilibrium conditions for loanable-funds (**22**), money (**23**), and finance (implicit in (**23**)) that if a *ceteris paribus* increase in thriftiness does not have a concomitant *direct* effect on the supply of money by banks ($\boldsymbol{m^s}$) or on the demands for money by households or firms ($\boldsymbol{f^s}$, $\boldsymbol{h^s}$, $\boldsymbol{f^f}$, $\boldsymbol{h^f}$, $\boldsymbol{h^c}$, $\boldsymbol{f^i}$, $\boldsymbol{h^p}$, $\boldsymbol{f^p}$, or $\boldsymbol{f^y}$), then the only way in which it can relieve the congestion in the capital market is through an effect on income that reduces the transactions or precautionary demands for money ($\boldsymbol{H^f}$, $\boldsymbol{H^c}$, $\boldsymbol{H^p}$, $\boldsymbol{F^p}$, and $\boldsymbol{F^y}$) in such a way as to increase the supply of speculative balances (**25**). And the only way an increase in saving (i.e., an increase in the *propensity to save*) can prevent the rate of interest from increasing in this situation, is by causing a fall in income that is sufficient to shift the demand for money (**20**), the supply and demand for loanable funds (**19**) and (**8**), and the supply of speculative balances (**25**) from **D***,

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[12] Cf., Keynes:

> It is possible, then, that confusion has arisen between credit in the sense of 'finance,' credit in the sense of 'bank loans' and credit in the sense of 'saving'.... It should be observed that a confusion between the first and the last would be one between a flow and a stock. Credit, in the sense of 'finance,' looks after a flow of investment. It is a revolving fund which can be used over and over again.... The same 'finance' can tackle one investment after another. But credit, in Prof. Ohlin's sense of 'saving,' relates to a stock. Each new net investment has new net saving attached to it. The saving can be used once only. It relates to the net addition to the stock of actual assets. (June 1937, p. 247)

Within this context, 'finance' obviously means *money* available to finance expenditures which is part of the revoliving fund of money in circulation.

S* and **D*** in **Figure** is by causing a fall in income **2** back to **D**, **S**, and **D**. This is the essence of Keynes' objection to Robertson's (1938) insistence that an increase in saving will relieve the congestion in the capital market in response to an increase in planned investment expenditures:

> The ... transition from a lower to a higher scale of activity involves an increased demand for liquid resources which cannot be met without a rise in the rate of interest, unless the banks are ready to lend more cash or the rest of the public to release more cash at the existing rate of interest. If there is no change in the liquidity position, the public can save ex-ante and ex-post and ex-anything-else until they are blue in the face, without alleviating the problem in the least—unless, indeed, the result of their efforts is to lower the scale of activity to what it was before. (1937b, p. 668-69)

At the same time, it must be noted that just as an increase in the propensity to save cannot *in itself* have a *direct* effect on the rate of interest in Keynes' general theory and, therefore, cannot *in itself* relieve the congestion in the capital market, an increase in the propensity to save also cannot *in itself* have a *direct* effect on income as defined by *the value of output produced*. The concomitant decrease in the demand for consumption goods will cause the proceeds received from the sale of consumption goods to fall which, *if it persists*, must eventually cause a change in *expectations* with regard to the proceeds to be obtained from the production and sale of additional consumption goods. These changed expectations must, in turn, *cause* a reduction in employment, output, and income in the consumption-goods industries. *It is only by setting this causal chain of events in motion through time* that an increase in the propensity to

save can have an effect on the value of output produced in the consumption-goods industries and, thereby, decrease the active demand for cash associated with income in this *ceteris paribus* situation. It is the fall in income that will have the effect of decreasing the active demand for cash and thereby increasing the *supply* of speculative balances in this situation, not the increase in saving itself. And it is the increase in the *supply* of speculative balances that will force wealth holders to choose between a lower rate of interest or increasing their holdings of speculative balances as an asset and force banks to choose between accepting a lower rate of interest or increasing their reserves that is the *direct* cause of the resulting fall in the rate of interest, not the increase in thriftiness itself. (Keynes, 1936, pp. 52-63; Hayes; Blackford, 2020a, pp. 20-77; 2020b, chs. 2-4)

Finally, it must be noted that this explanation as to how an increase in thriftiness can lead to a fall in the rate of interest through a change in expectations and a subsequent fall in employment, output, and income which, in turn, decreases the transactions demand for money and creates an increase in the supply of speculative balances that *causes* the rate of interest to fall is *dynamic* and is explicitly stated in *causal* terms. (Blackford, 2020a; 2020b, chs. 2, 3, 6)

III. Robertson's Definition of Income

The significance of the difference between Robertson's and Keynes' definitions of income within the context of Keynes' general theory can be seen by replacing Keynes' (1936, pp. 52-65) definition of income as the value of output produced in the model specified above with Robertson's definition of income as the value of output sold. (Robertson, 1933a, pp. 401-2,

1933b, pp. 710-1, 1936, p. 171; Hawtrey, pp. 702, 704) Since the rates of consumption and investment expenditures (i.e., sales) are given by C_t and I_t in this model, Robertson's definition is easily incorporated by defining aggregate income Y_t as the sum of C_t and I_t:

(26) $Y_t = C_t + I_t$.

By virtue of this definition, the transactions and precautionary demands for money by households and firms H^c, H^p, F^i, F^p, and F^y as specified in (**11**), (**9**), (**2**), (**4**), and (**3**) are no longer simply direct functions of income Y_t, but now are direct functions of $C_t + I_t$, and the aggregate demands for money M_t^d (**20**) and loanable funds L_t^d (**8**) are also direct functions of $C_t + I_t$, while the aggregate supply of loanable funds L_t^s (**19**) and finance Q_t^s (**25**) become inverse functions of $C_t + I_t$. Only the supply of money M_t^s (**16**) is unaffected by Robertson's definition of income since the aggregate supply of money is not assumed to be a function of income Y_t. This means that it is no longer possible to consider a *ceteris paribus* change in C_t while holding both I_t and Y_t constant. As a result, in analyzing the effects of a *ceteris paribus* fall in C_t on the rate of interest we must decide whether this fall in C_t is accompanied by a fall in Y_t and I_t remains unchanged, or an increase in I_t and Y_t remains unchanged, or if both Y_t and I_t are going to change. Herein lies the source of much of the confusion that surrounded the liquidity-preference/loanable-funds controversy. (cf., Blackford, 2020b, ch. 4; Tsiang; Horwich; and Kohn.)

If we accept Robertson's definition of income (**26**) and consider a *ceteris paribus* situation in which C_t falls and Y_t remains unchanged, and, at the same

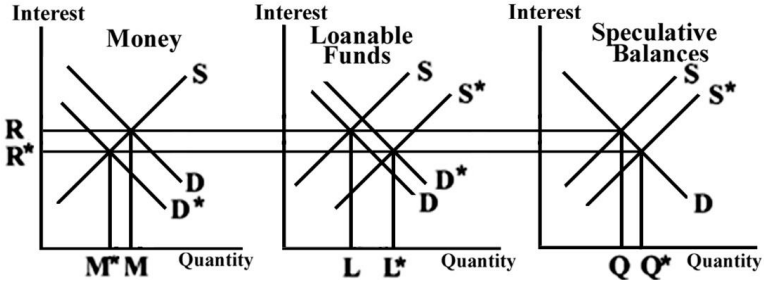
time, assume I_t increases so as to offset the effect of the fall in C_t on Y_t the results will be logically and mathematically the same as were obtained in examining **Figure 1** above since there will be no change in Y_t as given by (**26**) that results from the change in C_t. But if I_t is assumed to remain unchanged and Y_t is assumed to fall the results, as we shall see, will be entirely different. Thus, it would appear that the fundamental difference between Robertson and Keynes is a mere technicality relating to the kind of *ceteris paribus* analysis one chooses. The fallacious nature of this apparent technicality can be seen by accepting Robertson's definition of income (**26**) and considering the effects of an increase in thriftiness that takes the form of a *ceteris paribus* decrease in C_t and assumes I_t remains unchanged.

As was noted above, a *ceteris paribus* increase in thriftiness that leads to a fall in C_t must decrease the flow of money made available to firms through the consumption-goods market and, at the same time, increase the flow of money made available to firms through the loanable funds market. As we saw in examining **Figure 1**, if the supply and demand for money and the levels of employment, output, and *income* are held constant in this situation, firms must increase their demand for loanable funds in order to maintain their transactions and precautionary balances and households must increase the supply of loanable funds by the same amount in order to dispose of the excess balances they would otherwise accumulate. This is no longer the case if income Y_t as defined by (**26**) is allowed to fall and investment expenditures I_t (including inventories and the demand for money needed to maintain the current scale of operations)

are assumed to remain constant.

Given Robertson's definition of income (**26**) if the flow of investment expenditures I_t is assumed to remain constant a decrease in C_t must be accompanied by an equal decrease in the flow of income Y_t. This changes the behavior of the model dramatically. The fall in income Y_t will decrease the demand for money (**20**) while the demand (**8**) and the supply (**19**) of loanable funds will increase along with the supply of speculative balances (**25**); only the supplies of money (**16**) and speculative balances (**24**) will remain unchanged. This situation is illustrated in **Figure 3** where **S**, **D**, **R**, **L**, **M**, and **Q** are the initial curves and values as defined in **Figure 1**.

Figure 3: Robertson's Increase in Thriftiness.



The fall in the demand for money is represented in this figure by the shift in the money demand curve from **D** to **D***; the increase in the supply of loanable funds and speculative balances is represented by the shift in their supply curves from **S** to **S***, and the increase in the demand for loanable funds by the shift in the demand for loanable funds curve from **D** to **D***.

Since a fall in Y_t cannot have a direct effect on the supply of money (**16**) or demand for speculative balances (**24**) in the *ceteris paribus* situation examined in this figure the result is a) a fall in the equilibrium

rate of interest from **R** to **R***, b) an increase in borrowing from **L** to **L***, c) a decrease in the equilibrium quantity of money from **M** to **M***, and d) an increase in the amount of money held for speculative purposes from **Q** to **Q***.

The results in **Figure 3** are diametrically opposed to those of **Figure 1**, and they are formally correct given a) equations (**1**) - (**25**), b) the *assumption* that income $\boldsymbol{Y_t}$ is defined by sales as given by (**26**), and c) the assumption that $\boldsymbol{I_t}$ remains unchanged as $\boldsymbol{C_t}$ falls. Unfortunately, making these *assumptions* does not change the fact that it is impossible to give a *logically consistent, causal explanation* as to how these results are supposed to come about in the real world without taking into consideration the effects of changes in expectations, employment, output, and the value of output produced.

What happens to the value of output produced as $\boldsymbol{Y_t}$ falls in **Figure 3**? If it remains unchanged, how is it financed? If the value of output produced falls, why does it fall? Is it because expectations are unit-elastic and producers adjust output *instantaneously* to changes in sales? (Blackford, 2020a, pp. 20-8; 2020b, ch. 3) If expectations are not unit-elastic, what *causes* the value of output produced to fall in a situation in which producers expect to be able to maintain their scale of operations today and sell at a profit tomorrow? If the expectations of producers do adjust instantaneously to changes in sales such that investment does not change in response to a fall in consumption, why does this instantaneous change in expectations leave investment unchanged? If investment does change, will it increase or decrease and why? (Blackford, 1018b, 2020b)

There is no reason to believe that a fall in sales

can, *in itself*, cause a fall in the transactions demand for money in the absence of a change in expectations with regard to the *profitability* of producing at the current level of employment and output *no matter how we define income*. Until employment, output, and the value of output produced fall in this situation *there is no reason to expect a change in the volume of transactions*, and in the absence of a change in the volume of transactions there is no reason to expect a change in the demand for money or the rate of interest. Assuming the demand for money is a function of sales instead of the value of output produced does not change this *reality* irrespective of the results in **Figure 3**. (Blackford, 2020a; 2020b; Hawtrey, p. 702-4; and cf., Tsiang; Robertson; Kohn)

It is, of course, reasonable to argue that the reduction in sales, *if it persists*, must cause a change in expectations that must lead to a fall in employment, output, income, and the volume of transactions *through time*, and that the result will be a shift in the curves in **Figure 3**. (Blackford, 1018, 2020b, chs. 2, 3, 6) But it is *not* reasonable to confuse *the temporal order in which events must occur* and insist that an increase in saving "lowers the rate of interest quite directly through swelling the money stream of demand for securities" (Robertson, 1940, p. 19) as if saving is the direct cause of the fall in the rate of interest rather than a change in expectations that causes employment, output, and income to fall thereby *causing* a decrease in the demand for transactions balances that increases the *supply* of speculative balances. (Blackford, 2020b, chs. 2, 3, 6) This is particularly so in light of the fact that the fall in income can be expected to *cause* a reduction in the *flow* of savings and thereby reverse the "swelling ... money stream of demand for

securities" as the rate of interest falls. (Blackford, 2020a, p. 174-5)

Nor is it reasonable to ignore the effects of the negative change in expectations that is the direct *cause* of the fall in output on the demand for investment goods and argue that the resulting fall in the rate of interest will lead to an increase in the accumulation of capital. (Blackford, 2020a, pp. 78-94; 2020b, chs. 1, 3) And it is just plain foolish to ignore the effects of a fall in consumption on prospective yields and implement policies that have the effect of increasing the propensity to save in the belief that all that is necessary to avoid Keynes' long-period problem of saving is "a progressive increase in the supply of money" (Robertson, 1936, p. 189) in the face of speculative bubbles, an increasing concentration of income, increasing trade deficits, and a dramatic increase in debt relative to income as was the case leading up to the Crash of 2008. (Blackford, 2018; 2020a, 2020b, ch. 1)

List of Equations

A-a. A Model of Keynes' Theory of Interest

Firms

(1) $F_t^f = f^f(R_t), \qquad f^{f'} < 0.$

(2) $F_t^i = f^i(Y_t), \qquad f^{i'} > 0,$

(3) $F_t^y = f^y(Y_t), \qquad f^{y'} > 0.$

(4) $F_t^p = f^p(Y_t), \qquad f^{p'} > 0,$

(5) $F_t^s = f^s(R_t), \qquad f^{s'} < 0.$

(6) $F_t^{md} = F_t^f + F_t^i + F_t^y + F_t^p + F_t^s$

$= f^f(R_t) + f^i(Y_t) + f^y(Y_t) + f^p(Y_t) + f^s(R_t).$

(7) $F_t^n = F_{t-x} + \int_{t-x}^{t}\left((C_t + I_t + T_t^n) - (Y_t^p + I_t)\right)dt, \; x > 0,$

$= F_{t-x} + \int_{t-x}^{t}\left(C_t + T_t^n - Y_t^p\right)dt.$

(8) $L_t^d = \left(F_t^f + F_t^i + F_t^y + F_t^p + F_t^s\right) - F_t^n$

$= \left(f^f(R_t) + f^i(Y_t) + f^y(Y_t) + f^p(Y_t) + f^s(R_t)\right) -$

$\left(F_{t-x} + \int_{t-x}^{t}\left(C_t + T_t^n - Y_t^p\right)dt\right).$

Households

(9) $H_t^p = h^p(Y_t), \qquad h^{p'} > 0,$

(10) $H_t^s = h^s(R_t), \qquad h^{s'} < 0.$

(11) $H_t^c = h^c(Y_t), \qquad h^{c'} > 0.$

(12) $H_t^f = h^f(R_t), \qquad h^{f'} < 0.$

(13) $H_t^{md} = H_t^p + H_t^s + H_t^c + H_t^f$

$= h^p(Y_t) + h^s(R_t) + h^c(Y_t) + h^f(R_t).$

(14) $H_t^n = H_{t-x} + \int_{t-x}^{t}\left(Y_t^p - C_t - T_t^n\right)dt, \; x > 0.$

(15) $L_t^{hs} = H_t^n - (H_t^p + H_t^s + H_t^c + H_t^f)$

$= \left(h^p(Y_t) + h^s(R_t) + h^c(Y_t) + h^f(R_t)\right).$

Banks

$$(16)\quad M_t^s = m^s(R_t), \qquad m^{s'} > 0.$$

$$(17)\quad L_t^{bs} = M_t^s - M_{t-x} = m^s(R_t) - M_{t-x},$$

$$(18)\quad M_{t-x} = F_{t-x} + H_{t-x}.$$

Equilibrium Conditions

$$(19)\quad L_t^s = (M_t^s - M_0) + \left((H_0 + H_t^n) - \left(H_t^p + H_t^s + H_t^c + H_t^f\right)\right)$$
$$= \left(m^s(R_t) - M_{t-x}\right) + \left(\left(H_{t-x} + \int_{t-x}^{t}\left(Y_t^p - C_t - T_t^n\right)dt\right) - \left(h^p(Y_t) + h^s(R_t) + h^c(Y_t) + h^f(R_t)\right)\right).$$

$$(20)\quad M_t^d = \left(F_t^p + H_t^p + F_t^s + H_t^s\right) + \left(\left(F_t^f + H_t^f\right) + \left(H_t^c + F_t^i + F_t^y\right)\right)$$
$$= \left(f^p(Y_t) + h^p(Y_t) + f^s(R_t) + h^s(R_t)\right) + \left(\left(f^f(R_t) + h^f(R_t)\right) + \left(h^c(Y_t) + h^c(Y_t) + f^i(Y_t) + f^y(R_t)\right)\right).$$

$$(21)\quad \left(F_t^f + F_t^i + F_t^y + F_t^p + F_t^s\right) - \left(F_{t-x} + F_t^n\right) = \left(M_t^S - M_{t-x}\right) + \left(\left(H_{t-x} + H_t^n\right) - \left(H_t^p + H_t^s + H_t^c + H_t^f\right)\right)$$
$$\left(f^f(R_t) + f^i(Y_t) + f^y(Y_t) + f^p(Y_t) + f^s(R_t)\right) - \left(F_{t-x} + \int_{t-x}^{t}\left(C_t + T_t^n - Y_t^p\right)dt\right)$$
$$= \left(\left(m^s(R_t) - M_{t-x}\right) + \left(H_{t-x} + \int_{t-x}^{t}\left(Y_t^p - C_t - T_t^n\right)dt\right)\right) - \left(h^p(Y_t) + h^s(R_t) + h^c(Y_t) + h^f(R_t)\right),$$

$$(22)\quad \left(F_t^f + F_t^i + F_t^y + F_t^p + F_t^s\right) = M_t^S - \left(H_t^p + H_t^s + H_t^c + H_t^f\right),$$
$$\left(f^f(R_t) + f^i(Y_t) + f^Y(Y_t) + f^p(Y_t) + f^s(R_t)\right) = m^s(R_t) - \left(h^p(Y_t) + h^s(R_t) + h^c(Y_t) + h^f(R_t)\right).$$

$$(23)\quad M_t^S = \left(F_t^p + H_t^p + F_t^s + H_t^s\right) + \left(\left(F_t^f + H_t^f\right) + \left(H_t^c + F_t^i + F_t^y\right)\right)$$

$$m^s(R_t) = \left(f^p(Y_t) + h^p(Y_t) + f^s(R_t) + h^s(R_t)\right) + \left(\left(f^f(R_t) + h^f(R_t)\right) + \left(h^c(Y_t) + h^c(Y_t) + f^i(Y_t) + f^y(R_t)\right)\right)$$

$$(24)\quad Q_t^d = F_t^s + H_t^s = f^s(R_t) + h^s(R_t),$$

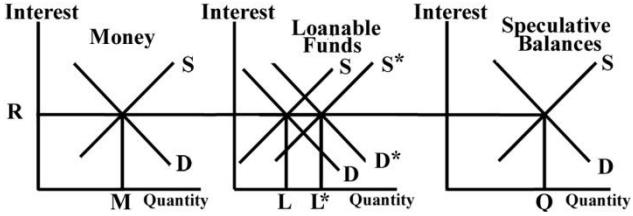
$$(24)\quad Q_t^d = F_t^s + H_t^s = f^s(R_t) + h^s(R_t),$$

$$(25)\quad Q_t^s = M_t^s - \left(F_t^i + F_t^f + H_t^c + H_t^f + F_t^y + F_t^p + H_t^p\right) = m^s(R_t) - \left(f^i(Y_t) + f^f(R_t) + h^c(Y_t) + h^f(R_t) + f^y(Y_t) + f^p(Y_t) + h^p(Y_t)\right)$$

A-b. Thriftiness, Investment, and Finance

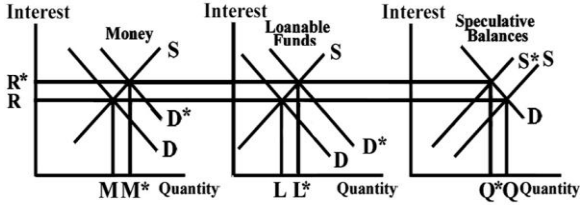
An Increase in Thriftiness

Figure 1: An Increase in Thriftiness.



The Demand for Finance

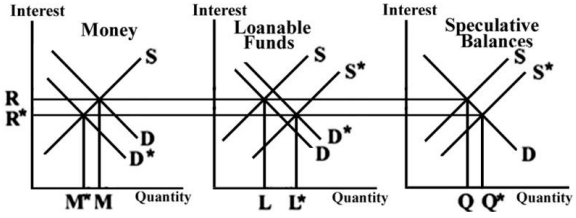
Figure 2: An Increase in Demand for Finance.



A-c. Robertson's Definition of Income

(26) $Y_t = C_t + I_t$.

Figure 3: Robertson's Increase in Thriftiness.



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