Mr. Keynes and the Neoclassics: A Reinterpretation

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Abstract

A Marshallian model of Keynes' integration of monetary and value theory that incorporates the prices of non-debt assets as well as the prices of consumption and investment goods is specified. This model is used to explain how short- and long-run equilibriums are obtained within the context of Keynes' general theory. It is demonstrated that the Keynesian IS/LM model is a special (static) case of the model specified in this paper and that the Marshallian roots of Keynes' model makes a logically consistent, causal analysis of dynamic behavior possible while the Walrasian roots of neoclassical models only allow for a description of dynamic behavior without explanation other than through the invocation of a mythical auctioneer.

Keywords: Macroeconomics, Monetary Theory, Causality, Methodology, Neoclassical Economics, Keynes, Keynesian Economics, Economic Stagnation, History of Economic Thought.

JEL Codes: B22, B41, D51, E12, E13, E41, E43, J23, Q11

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors, and the author has no conflicts of interest in this paper.

1. Introduction

It is well known that by virtue of the simultaneity assumption implicit in the Walrasian budget constraint (i.e., that all decisions are made simultaneously consistent with that constraint) a logically consistent, causal analysis of dynamic behavior is impossible in Walrasian models; it is the Walrasian *tâtonnement*/re-contract auctioneer that 'causes' prices and quantities to change within the Walrasian paradigm not decision-making units that actually exist in the real world. As a result, such models cannot be used to establish the temporal order in which events must occur—that is, the order in which endogenous variables must change in response to a change in an exogenous variable—in terms of the actual behavior of decision-making units. (Blackford 1975; 1976; Clower, Leijonhufvud, Minsky) It is the purpose of this paper to show that this is not the case when Keynes' integration of monetary and value theory is viewed from the perspective of the Marshallian paradigm.

A Marshallian model of Keynes' integration of monetary and value theory that incorporates the prices of non-debt assets as well as the prices of consumption and investment goods is specified below. The fundamental difference between the structure of this model and those of neoclassical economics is that the behavioral equations are assumed to be consistent with Marshallian supply and demand functions rather than the Walrasian supply and demand functions assumed by neoclassical economists. They are presumed to be determined by the optimizing behavior of decision-making units as they interact in markets, just as Walrasian supply and demand functions are presumed to be determined by optimizing behavior. The difference is that in Keynes' understanding of these functions they are specified by isolating those factors that are perceived to have a *direct* effect on the willingness of buyers and sellers to buy and sell in individual markets at any point in time whether the system is in equilibrium or not without assuming that these choices are constrained by an arbitrary Walrasian budget constraint. Instead, they are derived by observing the actual behavior of decision-making units in markets, hypothesizing with regard to the motivations of these units given their *expectations* with regard to those magnitudes that affect their choices *directly* in each individual market, and then reasoning through the logical implications of what the actual choices available to decision-making units and their motivations and expectations imply with regard to their willingness to buy and sell in individual markets. (cf., Blackford 1975; 1976; Clower; Lavoie and Godley) As a result, even though the set of equilibrium conditions specified when taken together define a general equilibrium of the system as a whole they are the product of a *partial* equilibrium analysis of individual markets in which *the values of individual variables are assumed to be determined by the choices of those decision-making units that actually have the power to determine the value of each variable at each point in time as the system evolves through time*.[1]

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[1] See Marshall (1920; 1961, Books III-IV) and cf., Keynes (1936, Books III-IV), Hayes, Brady, and Lavoie and Godley. It should, perhaps, be noted that the supply and demand for loanable funds are not independent behavioral functions in this model. In a world in which decision-making units do not go into debt simply for the sake being in debt—*a world in which debt, in itself, offers no satisfaction or utility*—there must be a demand for *money* for some reason other than the satisfaction of being in debt *before* there can be a willingness to borrow money. Thus, it is assumed that decision-making units borrow *money* only to meet their financial needs for *money*. Similarly, it is assumed that decision-making units lend money only to dispose

This model is used to explain how short- and long-run equilibriums are obtained within the context of Keynes' general theory, and how Keynes' Marshallian model is related to the Walrasian models of Neo-Classical economics. It is demonstrated that when Keynes' general theory is viewed from the perspective of Marshall, rather than Walras, it is possible to establish the temporal order in which events must occur in terms of the actual behavior of decision-making units as these units respond to changes in the exogenous determinants of the variables in each sector of the economy, and that, as a result, the Marshallian roots of Keynes' model make a logically consistent, causal analysis of dynamic behavior possible, while the Walrasian roots of neoclassical models only allow for a description of dynamic behavior without explanation other than through the invocation of a mythical auctioneer

2. Keynes' Macroeconomic Model

Keynes summarized the independent and dependent variables in the analytical framework he developed throughout *The General Theory* at the beginning of Chapter 18. He then outlined the aggregate model embodied in this framework as follows:

> Thus we can sometimes regard our ultimate independent variables as consisting of (1) the three fundamental psychological factors, namely, the psychological propensity to consume, the psychological attitude to liquidity and the psychological expectation of future yield from capital-assets, (2) the wage-unit as determined by the bargains reached between employers and employed, and (3) the quantity of money as determined by the action of the central bank; so that, if we take as given the factors specified above, these variables determine the national income (or dividend) and the quantity of employment. But these again would be capable of being subjected to further analysis, and are not, so to speak, our ultimate atomic independent elements [emphasis added]. (1936, p. 246-7)

In order to express the behavioral equations of the model described in this paragraph in the terminology used by Keynes throughout *The General Theory* it is necessary to define the model's variables in terms of Keynes' chosen units of measurement. It is also necessary to incorporate the crucial role played by *expectations* in Keynes' general theory in determining *causality* as the system changes through time. (Blackford 2022)

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of excess money balances they have no use for otherwise or in the case of trade credit to facilitate current transactions by providing for the payment of *money* at a later date. This makes the supply and demand for loanable funds *ex post* functions, determined within the system by the suppliers and demanders for money as dictated by the desired transactions of decision-making units as embodied in the behavioral equations of the model. As a result, there are no redundant equations in the model, and Walras' Law—*a law that can only be enforced by a mythical auctioneer*—has no relevance in this model, just as it has no relevance in Keynes' general theory or in the real world. (Blackford 1975; 1976; Clower) It should also be noted that this does not mean that the system-wide consistency requirements that Lavoie and Godley (p. 14) examined are violated. It only means that there is no reason to believe that *at any given point in time* the excess demands in the model sum to zero. As Lavoie and Godley have noted: "the λ parameters will be shifting around like mad, as people change opinions on what is appropriate, but they are always subject to the adding-up constraints." (p. 144)

2.1 Behavioral Equations

We begin by specifying the behavior equations of Keynes' model in terms of Keynes' chosen units of measurement. Keynes made use of but two units of measurement: money and the labor-unit where Keynes defined the labor-unit as "an hour's employment of ordinary labour" (1936, p. 41). In specifying the behavioral equation of Keynes' model in terms of money and labor-units we begin by a) dividing the money values of the rates at which consumption ($\boldsymbol{P_t^c C_t}$) and investment ($\boldsymbol{P_t^i I_t}$) goods are produced by what Keynes defined as the wage-unit ($\boldsymbol{W_t}$)—that is, "the money-wage of a labour-unit" (1936, p. 41)—in order to express the *rates* at which consumption ($\boldsymbol{C_t}$) and investment ($\boldsymbol{I_t}$) goods are produced in 'wage-units'—that is, in hours of ordinary labor per unit of time—and b) summing to obtain the aggregate level of output/income ($\boldsymbol{Y_t^w}$) measured in wage-units:[2]

$$\begin{aligned} \boldsymbol{Y_t^w} &= \boldsymbol{\frac{P_t^c C_t}{W_t} + \frac{P_t^i I_t}{W_t}} \\ &= \boldsymbol{C_t^w + I_t^w} \end{aligned} \qquad (\mathbf{1})$$

where $\boldsymbol{C_t^w}$ and $\boldsymbol{I_t^w}$ are the aggregate *rates* of consumption $\boldsymbol{C_t}$ and investment $\boldsymbol{I_t}$ goods production expressed in terms of hours of ordinary labor per unit of time and $\boldsymbol{P_t^c}$ and $\boldsymbol{P_t^i}$ are the (weighted average) prices of $\boldsymbol{C_t}$ and $\boldsymbol{I_t}$, respectively.[3] Similarly, the "psychological attitude to liquidity" is embodied in Keynes' liquidity-preference/money-demand function and is assumed to be an inverse function of the rate of interest ($\boldsymbol{R_t}$) and a direct function of income measured in wage-units $\boldsymbol{Y_t^w}$:

$$\boldsymbol{M_t^{wd} = m^d(Y_t^w, R_t), \quad m_1^d > 0, \;\; m_2^d < 0} \qquad (\mathbf{2})$$

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[2] It is rather misleading to refer to Keynes' wage-unit measure as "constant-wage-unit dollars" as Hansen did in 1953 (p. 44). When the value of a flow variable such as consumption, investment, income, or labor which is measured in money-units/time-unit (e.g., dollars/year) is divided by Keynes' wage-unit which is measured in money-units/labor-unit (e.g., dollars/hour-of-ordinary-labor) the money-units cancel and we are left with labor-units/time-unit or hours of ordinary labor per unit of time (i.e., (dollars/year)/(dollars/hour-of-ordinary-labor) = hour(s)-of-ordinary-labor/year). Similarly, when a stock variable such as debt, non-debt assets, or money which is measured in money-units (e.g., dollars) is divided by Keynes' wage-unit (e.g., dollars/hour-of-ordinary-labor) the money-units again cancel and we are left with labor-units or hours of ordinary labor (i.e., dollars/(dollars/hour-of-ordinary-labor) = hour(s)-of-ordinary-labor). As a result, money-units, constant or otherwise, cancel and are not part of the unit of measurement when a quantity is measured in 'wage-units'. It must be noted, however, that since Keynes' measure presumes that "different grades and kinds of labour … enjoy a more or less fixed relative remuneration" it is not clear that Keynes' measure is superior to the constant-dollar measure of neoclassical economics in light of the changes in relative remunerations for various kinds of labor that have occurred over the past fifty-odd years. See Piketty (Chap. 9).

[3] This specification does not presume homogeneous consumption or investment goods: any number of consumption and investment goods can be aggregated in this way simply by dividing the sum of the money values of the individual rates of consumption and investment goods production by the wage-unit $\boldsymbol{W_t}$ to obtain the hours of ordinary labor per unit of time that correspond to the money value of the aggregate.

where $\boldsymbol{M_t^{wd}}$ is the nominal value of the stock of money demanded $\boldsymbol{M_t^d}$ divided by the wage-unit $\boldsymbol{W_t}$ which expresses the value of the stock of money demanded $\boldsymbol{M_t^{wd}}$ in terms of hours of ordinary labor.[4]

Keynes assumed the stock of money to be exogenously "determined by the action of the central bank" in his summary above, but in 1937 he observed that "an illuminating way of expressing the liquidity-theory" is in terms of the willingness for the public "to become more or less liquid and … the banking system … to become more or less unliqulid." (1937b, p. 666) Hence, it is assumed that the quantity of money supplied by the financial system, measured in wage-units ($\boldsymbol{M_t^{ws}}$), is directly related to the rate of interest $\boldsymbol{R_t}$:

$$\boldsymbol{M_t^{ws} = m^s(R_t), \quad m^{s'} > 0} \qquad (\mathbf{3})$$

where $\boldsymbol{M_t^{ws}}$ is the nominal value of the stock of money in existence ($\boldsymbol{M_t}$) divided by the money wage $\boldsymbol{W_t}$ which yields the stock of money supplied expressed in terms of hours of ordinary labor.

It is assumed that the existing stock of non-debt assets ($\boldsymbol{A_t}$) is exogenously determined. This stock, measured in wage-units ($\boldsymbol{A_t^{ws}}$), is given by:

$$\boldsymbol{A_t^{ws} = \frac{P_t^a A_t}{W_t}} \qquad (\mathbf{4})$$
$$\boldsymbol{= A_t^w}$$

where $\boldsymbol{P_t^a}$ is the (weighted average) price of non-debt assets. The demand for non-debt assets measured in wage-units ($\boldsymbol{A_t^{wd}}$) is assumed to be inversely related to the price of non-debt assets $\boldsymbol{P_t^a}$ and the rate of interest $\boldsymbol{R_t}$ and directly related to output/income $\boldsymbol{Y_t^w}$:

$$\boldsymbol{A_t^{wd} = a^d(P_t^a, R_t, Y_t^w), \quad a_1^d, a_2^d < 0, \quad a_3^d > 0.} \qquad (\mathbf{5})$$

It is also instructive, for expository purposes, to specify the *non-debt asset equilibrium function* in this model even though Keynes did not utilize this relationship. This function can be obtained by setting the supply of non-debt assets (**4**) equal to the demand for non-debt assets (**5**),

$$\boldsymbol{A_t^w = a^d(P_t^a, R_t, Y_t^w)} \qquad (\mathbf{6})$$

and solving for the equilibrium price of non-debt assets $\boldsymbol{P_t^a}$ as a function of the stock of non-debt assets $\boldsymbol{A_t^w}$, rate of interest $\boldsymbol{R_t}$, and rate of output/income $\boldsymbol{Y_t^w}$:

$$\boldsymbol{P_t^a = a(A_t^w, R_t, Y_t^w), \quad a_1, a_2 < 0, \;\; a_3 > 0.} \qquad (\mathbf{7})$$

The "psychological propensity to consume" is embodied in Keynes' consumption functions.

[4] Keynes argued in *The General Theory* (p. 304) that the demand for money is a function of effective demand $\boldsymbol{Y_t^{we}}$ (**22**), and in his 1938 attempt to clarify the nature of the demand for money and its relationship to 'finance' Keynes also argued that the demand for money "is a function of income and of business habits" (1938, p. 321-2). I believe that the best way to incorporate this aspect of Keynes understanding of the demand for money is to assume that the demand for money is a direct function of realized output/income $\boldsymbol{Y_t^w}$ (**1**) and that changes in effective demand $\boldsymbol{Y_t^{we}}$ (**22**) have the effect of shifting the demand for money function $\boldsymbol{m^d(Y_t^w, R_t)}$ (**2**) by way of changes in the demand for 'finance'. See Bibow, Davidson, and Keynes (1937b).

If it is assumed that the supply price of consumption goods ($\boldsymbol{P_t^{cs}}$) is a direct function of the rate of consumption goods production $\boldsymbol{C_t^w}$ the supply price of consumption goods $\boldsymbol{P_t^{cs}}$ can be written as:[5]

$$\boldsymbol{P_t^{cs} = c^{sp}(C_t^w), \qquad c^{sp'} > 0.} \tag{8}$$

If it is also assumed that the demand price of consumption goods ($\boldsymbol{P_t^{cd}}$) is an inverse function of the rate of consumption goods production $\boldsymbol{C_t^w}$ and a direct function of the level of income $\boldsymbol{Y_t^w}$ the demand price of consumption goods $\boldsymbol{P_t^{cd}}$ can be written as:

$$\boldsymbol{P_t^{cd} = c^{dp}(C_t^w, Y_t^w), \;\; c_1^{dp} < 0, \; c_2^{dp} > 0.} \tag{9}$$

Keynes' consumption function can then be obtained by equating the supply price of consumption goods $\boldsymbol{P_t^{cs}}$ (**8**) and the demand price of consumption goods $\boldsymbol{P_t^{cd}}$ (**9**) to obtain:

$$\boldsymbol{c^{sp}(C_t^w) = c^{dp}(C_t^w, Y_t^w) = P_t^c} \tag{10}$$

and solving this equation for the rate of consumption goods demanded ($\boldsymbol{C_t^{wd}}$) as a function of income $\boldsymbol{Y_t^w}$:

$$\boldsymbol{C_t^{wd} = c(Y_t^w), \quad 0 < c' < 1} \tag{11}$$

where $\boldsymbol{c}$ denotes Keynes' aggregate consumption function, $\boldsymbol{C_t^{wd}}$ is the rate of consumption goods demanded at each level of income $\boldsymbol{Y_t^w}$ given the (weighted average) price of consumption goods $\boldsymbol{P_t^c}$ as determined by suppliers and demanders in the markets for consumption goods, and it is assumed that the Marginal Propensity to Consume (MPC) $\boldsymbol{c'}$ lies between zero and one. Thus, Keynes' aggregate savings function ($\boldsymbol{s}$) is given by:

$$\begin{aligned} \boldsymbol{S_t^w} &= \boldsymbol{Y_t^w - c(Y_t^w)} \\ &= \boldsymbol{s(Y_t^w), \quad 0 < s' < 1} \end{aligned} \tag{12}$$

where $\boldsymbol{S_t^w}$ is the rate at which income $\boldsymbol{Y_t^w}$ is not spent on consumption goods.

The "psychological expectation of future yield from capital assets" is embodied in Keynes' Marginal Efficiency of Capital (MEC) schedule which Keynes defined as: "The relation between the prospective yield of a capital-asset and its supply price or replacement cost." (1936, pp. 135-6) This schedule can be obtained in a manner parallel to the derivation of Keynes' consumption function. If it is assumed that the supply price of investment goods ($\boldsymbol{P_t^{is}}$) is a direct function of the rate of investment goods production measured in wage-units ($\boldsymbol{I_t^w}$) the supply price of investment goods $\boldsymbol{P_t^{is}}$ can be written as:

$$\boldsymbol{P_t^{is} = i^{sp}(I_t^w), \qquad i^{sp'} > 0.} \tag{13}$$

If it is also assumed that the demand price of investment goods ($\boldsymbol{P_t^{id}}$) is an inverse function of the rate of investment goods production $\boldsymbol{I_t^w}$ and the rate of interest $\boldsymbol{R_t}$ and a direct function of the price of non-debt assets $\boldsymbol{P_t^a}$ (Keynes 1936, p. 151) and the demand for consumption goods $\boldsymbol{c(Y_t^w)}$ (Keynes 1936, pp. 46, 210-12), the demand price of investment goods $\boldsymbol{P_t^{id}}$ can be written

[5] It is assumed that the supply-price functions $\boldsymbol{c^{sp}}$ (8) and $\boldsymbol{i^{sp}}$ (13) below are derived from their corresponding employment functions and, hence, are independent of industry output. See Keynes (1936, pp. 280-91).

as:

$$\boldsymbol{P_t^{id} = i^{dp}\left(I_t^w, R_t, P_t^a, c(Y_t^w)\right)} \qquad (\mathbf{14})$$
$$\boldsymbol{= i^{dp}(I_t^w, R_t, P_t^a, Y_t^w), \quad i_1^{dp}, i_2^{dp} < 0, \; i_3^{dp} \; i_4^{dp} > 0}$$

Keynes' MEC schedule can then be obtained by equating the supply price of investment goods $\boldsymbol{P_t^{is}}$ (**13**) and demand price of investment goods $\boldsymbol{P_t^{id}}$ (**14**) to obtain:

$$\boldsymbol{i^{sp}(I_t^w) = i^{dp}(I_t^w, R_t, P_t^a, Y_t^w) = P_t^i} \qquad (\mathbf{15})$$

and solving for Keynes' MEC schedule:

$$\boldsymbol{I_t^{wd} = i(R_t, P_t^a, Y_t^w), \qquad i_1 < 0, \quad i_2, i_3 > 0} \qquad (\mathbf{16})$$

where $\boldsymbol{i}$ denotes Keynes' MEC schedule, and $\boldsymbol{I_t^{wd}}$ is the rate at which investment goods are demanded at each rate of interest $\boldsymbol{R_t}$ given the level of output/income $\boldsymbol{Y_t^w}$ and the prices of investment goods $\boldsymbol{P_t^i}$ and non-debt assets $\boldsymbol{P_t^a}$ as determined by the suppliers and demanders in the markets for investment goods and non-debt assets.[6]

Equations (**11**) and (**16**) imply that Keynes' aggregate demand function can be written as:

$$\boldsymbol{Y_t^{wd} = C_t^{wd} + I_t^{wd}} \qquad (\mathbf{17})$$
$$\boldsymbol{= c(Y_t^w) + i(R_t, P_t^a, Y_t^w).}$$

where $\boldsymbol{Y_t^{wd}}$ is the aggregate demand for output measured in wage-units, and to simplify the exposition it is assumed that net income is a unique function of gross income.

To complete the behavioral equations in Keynes' aggregate model it is necessary to explain the relationship between the level of employment and *effective demand* where Keynes defined effective demand as the point at which the "entrepreneurs' expectation of profits will be maximized" (1936, p. 25). Keynes explained this relationship in Chapter 20 in terms of his employment function:

> In Chapter 3 we have defined the aggregate supply function $Z = \varphi(N)$, which relates the employment N with the aggregate supply price of the corresponding output. The employment function only differs from the aggregate supply function in that it is, in effect, its inverse function and is defined in terms of the wage-unit; the object of the employment function being to relate the amount of the effective demand, measured in terms of the wage-unit, directed to a given firm or industry or to industry as a whole with the amount of employment, the supply price of the output of which will compare to that amount of effective demand. Thus, if an amount of effective demand D_{wr}, measured in wage-units, directed to a firm or industry calls forth an amount of employment N_r in that firm or industry, the employment function is given by $N_r = F_r(D_{wr})$. Or, more generally, if we are entitled to assume that D_{wr} is a unique function of the total effective demand D_w,

[6] Cf., Keynes (1937a, pp. 217-8) and:

> There will be an inducement to push the rate of new investment to the point which forces the supply-price of each type of capital-asset to a figure which, taken in conjunction with its prospective yield, brings the marginal efficiency of capital in general to approximate equality with the rate of interest. (1936, p. 248)

the employment function is given by $N_r = F_r(D_w)$. (1936, p. 280)

Thus, if we assume the rates at which labor is demanded measured in wage-units in the investment- and consumption-goods industries are direct functions of the effective demands for investment ($\boldsymbol{I_t^{we}}$) and consumption ($\boldsymbol{C_t^{we}}$) goods, respectively, measured in wage-units in each of these industries—that is, are direct functions of the level of output at which producers *expect* to maximize their profits—we can write the demand for labor measured in wage-units in the investment-goods industries ($\boldsymbol{N_t^{wid}}$) as:

$$\boldsymbol{N_t^{wid} = n^{id}(I_t^{we}), \qquad n^{id'} > 0} \tag{18}$$

where the value of $\boldsymbol{N_t^{wid}}$ is expressed in terms of hours of ordinary labor per unit of time. Similarly, the demand for labor measured in wage-units in the consumption-goods industries ($\boldsymbol{N_t^{wcd}}$) is given by:

$$\boldsymbol{N_t^{wcd} = n^{cd}(C_t^{we}), \qquad n^{cd'} > 0.} \tag{19}$$

And "if we are entitled to assume that [employment in each firm or industry] is a unique function of the total effective demand" (**18**) and (**19**) imply that Keynes' aggregate employment function can be written as: [7]

$$\begin{aligned} \boldsymbol{N_t^{w}} &= \boldsymbol{n^{id}(Y_t^{we}) + n^{cd}(Y_t^{we})} \\ &= \boldsymbol{n(Y_t^{we}), \qquad n' = 1} \end{aligned} \tag{20}$$

where $\boldsymbol{Y_t^{we}}$ is the aggregate effective demand—that is, the aggregate level of output at which producers *expect* to maximize their profits—as given by the sum of $\boldsymbol{C_t^{we}}$ and $\boldsymbol{I_t^{we}}$:

$$\boldsymbol{Y_t^{we} = C_t^{we} + I_t^{we}.} \tag{21}$$

And since the "*employment function* only differs from the aggregate supply function in that it is, in effect, its inverse and is defined in terms of the wage-unit," the inverse of (**20**) yields Keynes' aggregate supply function defined in terms of wage-units:

$$\boldsymbol{Y_t^{ws} = n^{-1}(N_t^{w}), \quad n^{-1'} = 1} \tag{22}$$

"which relates the employment [$\boldsymbol{N_t^{w}}$] with the aggregate supply price [$\boldsymbol{Y_t^{ws}}$] of the corresponding output [$\boldsymbol{Y_t^{w}}$]" ($= \boldsymbol{Y_t^{we} = Y_t^{ws}}$) measured in wage-units. (cf., Brady) And since output $\boldsymbol{Y_t^{w}}$ is determined by effective demand $\boldsymbol{Y_t^{ed}}$ ($=\boldsymbol{Y_t^{w}}$) the inverse of Keynes' aggregate employment function (**20**) allows us to write Keynes' aggregate demand function (**17**) as:

$$\begin{aligned} \boldsymbol{Y_t^{wd}} &= \boldsymbol{c\left(n^{-1}(N_t^{w})\right) + i\left(R_t, P_t^{a}, n^{-1}(N_t^{w})\right)} \\ &= \boldsymbol{y^{d}(N_t^{w}, R_t, P_t^{a}), \quad y_1^{d}, y_3^{d} > 0,\ y_2^{d} < 0} \end{aligned} \tag{23}$$

where $\boldsymbol{Y_t^{wd}}$ denotes the aggregate demand at each level of employment $\boldsymbol{N_t^{w}}$, both measured in wage-units, given the rate of interest $\boldsymbol{R}$ and price of non-debt assets $\boldsymbol{P_t^{a}}$.

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[7] Since the *aggregate* employment function is defined in terms of wage-units *net of user cost*, both $\boldsymbol{N_t^{w}}$ and $\boldsymbol{Y_t^{we}}$ define the number of hours-of-ordinary-labor/time-unit needed to satisfy $\boldsymbol{Y_t^{we}}$. Thus, $\boldsymbol{N_t^{w}} = \boldsymbol{Y_t^{we}}$ and $\boldsymbol{dN_t^{w}} = \boldsymbol{dY_t^{we}}$ which implies that $\boldsymbol{n' = n^{-1'}} = 1$. See footnote 2 above and Keynes (1936, p. 55n).

2.2 Dynamic Adjustment Functions

In specifying the dynamic adjustment functions that determine the behavior of the individual variables in Keynes' aggregate model it is assumed that demanders and suppliers of money adjust the rate of interest $\boldsymbol{R_t}$ to equate the demand for money $\boldsymbol{M_t^{wd}}$ (**2**) to the supply of money $\boldsymbol{M_t^{w}}$ (**3**) measured in wage-units in accordance with what Leijonhufvud (2006, pp. 61-77) referred to as Marshall's "laws of motion", namely, that the suppliers and demanders for money adjust rate of interest $\boldsymbol{R_t}$ to equate the supply $\boldsymbol{M_t^{ws}}$ and demand $\boldsymbol{M_t^{wd}}$ for money:

$$\begin{aligned} \boldsymbol{dR_t} &= \boldsymbol{g^r}\left(\boldsymbol{M_t^{wd}} - \boldsymbol{M_t^{ws}}\right) \\ &= \boldsymbol{g^r}\left(\boldsymbol{m^d}(\boldsymbol{Y_t^w}, \boldsymbol{R_t}) - \boldsymbol{m^s}(\boldsymbol{R_t})\right) \end{aligned} \qquad (\mathbf{24})$$

and adjust the stock of money $\boldsymbol{M_t}$ to the short side of the market:

$$\begin{aligned} \boldsymbol{dM_t} &= \boldsymbol{g^m}\left(\boldsymbol{M_t^{wd}} - \boldsymbol{M_t^{w}}\right) \\ &= \boldsymbol{g^m}\left(\boldsymbol{m^d}(\boldsymbol{Y_t^w}, \boldsymbol{R_t}) - \boldsymbol{M_t^{w}}\right) \end{aligned} \qquad (\mathbf{25})$$

where $\boldsymbol{dR_t}$ and $\boldsymbol{dM_t}$ are the time derivative operator $\boldsymbol{d}$ (=$\boldsymbol{d/dt}$) applied to $\boldsymbol{R_t}$ and $\boldsymbol{M_t}$, and the time derivative functions $\boldsymbol{g^r}$ and $\boldsymbol{g^m}$ (as well as the time derivative functions specified below) are assumed to increase monotonically through the origin.[8] It is also assumed that demanders and suppliers of non-debt assets adjust the price of non-debt assets $\boldsymbol{P_t^a}$ to equate the demand for non-debt assets $\boldsymbol{A_t^{wd}}$ (**5**) to the existing stock of non-debt assets $\boldsymbol{A_t^{ws}}$ (**4**) measured in wage-units:

$$\begin{aligned} \boldsymbol{dP_t^a} &= \boldsymbol{g^{pa}}\left(\boldsymbol{A_t^{wd}} - \boldsymbol{A_t^{sw}}\right) \\ &= \boldsymbol{g^{pa}}\left(\boldsymbol{a^d}(\boldsymbol{P_t^a}, \boldsymbol{R_t}, \boldsymbol{Y_t^w}) - \boldsymbol{A_t^w}\right). \end{aligned} \qquad (\mathbf{26})$$

Next it is assumed that producers in the investment- and consumption-goods industries adjust their expectations to equate the effective demands for consumption $\boldsymbol{C_t^{we}}$ and investment $\boldsymbol{I_t^{we}}$ goods to the actual demands for these goods $\boldsymbol{C_t^{wd}}$ and $\boldsymbol{I_t^{wd}}$ as defined by the inverses of (**9**) and (**14**):

$$\begin{aligned} \boldsymbol{dC_t^{we}} &= \boldsymbol{g^{ce}}\left(\boldsymbol{C_t^{wd}} - \boldsymbol{C_t^{we}}\right) \\ &= \boldsymbol{g^{ce}}\left(\boldsymbol{c^{dp-1}}(\boldsymbol{P_t^c}, \boldsymbol{Y_t^w}) - \boldsymbol{C_t^{we}}\right) \end{aligned} \qquad (\mathbf{27})$$

$$\begin{aligned} \boldsymbol{dI_t^{we}} &= \boldsymbol{g^{ie}}\left(\boldsymbol{I_t^{wd}} - \boldsymbol{I_t^{we}}\right) \\ &= \boldsymbol{g^{ie}}\left(\boldsymbol{i^{dp-1}}\left(\boldsymbol{P_t^i}, \boldsymbol{R_t}, \boldsymbol{P_t^a}, \boldsymbol{Y_t^w}\right) - \boldsymbol{I_t^{we}}\right) \end{aligned} \qquad (\mathbf{28})$$

as they adjust the rates of consumption $\boldsymbol{C_t^w}$ and investment $\boldsymbol{I_t^w}$ goods production to their respective effective demands:

$$\boldsymbol{dC_t^w} = \boldsymbol{g^c}(\boldsymbol{C_t^{we}} - \boldsymbol{C_t^w}) \qquad (\mathbf{29})$$

[8] It should be noted that the time derivative functions in this model are not necessarily assumed to be continuous, well-behaved mathematical functions in the real world even though for ease of exposition they will be discussed as such here. They can be modified to fit the hypotheses of an endogenous money supply or markup pricing, etc., as one wishes, but to do so here is beyond the scope of this paper. Cf., Brady, Hayes, Lavoie and Godley, Wray, and Keynes (1936; 1937a).

$$dI_t^w = g^i(I_t^{we} - I_t^w) \quad (30)$$

and that suppliers and demanders in the markets for investment and consumption goods adjust the prices of investment P_t^i and consumption P_t^c goods to equate the supplies (I_t^s) and (C_t^s) and demands (I_t^d) and (C_t^d) for investment and consumption goods as given by the inverses of the supply prices P_t^{is} (**13**) and P_t^{cs} (**8**) and demand prices P_t^{id} (**14**) and P_t^{cd} (**9**) for these goods:

$$dP_t^i = g^{pi}(I_t^d - I_t^s) \quad (31)$$
$$= g^{pi}\left(i^{dp-1}(P_t^i, R_t, P_t^a, Y_t^w) - i^{sp-1}(P_t^i)\right)$$
$$dP_t^c = g^{pc}(C_t^d - C_t^s) \quad (32)$$
$$= g^{pc}\left(c^{dp-1}(P_t^c, Y_t^w) - c^{sp-1}(P_t^c)\right)$$

Finally, it is assumed that producers adjust aggregate employment (N_t^w) to equate the aggregate supply of output Y_t^{ws} (**22**) and the aggregate effective demand for output Y_t^{we} (**21**):[9]

$$dN_t^w = g^{nd}(Y_t^{we} - Y_t^{ws}) \quad (33)$$
$$= g^{nd}\left(Y_t^{we} - n^{-1}(N_t^w)\right)$$

as the effective demand for output Y_t^{we} (**21**) adjusts to the actual demand for output Y_t^{wd} (**23**) by way of the identity implied by (**21**), (**27**), and (**28**):

$$dY_t^{we} = dC_t^{we} + dI_t^{we} \quad (34)$$
$$= g^{ce}\left(c^{dp-1}(P_t^{cd}, Y_t^w) - C_t^{we}\right) + g^{ie}\left(i^{dp-1}(P_t^{id}, R_t, P_t^a, Y_t^w) - I_t^{we}\right)$$

and aggregate output/income Y_t^w adjusts by way of the identity implied by (**1**), (**29**), and (**30**):

$$dY_t^w = dC_t^w + dI_t^w \quad (35)$$
$$= g^c(C_t^{we} - C_t^w) + g^i(I_t^{we} - I_t^w).$$

2.3 Structure of Keynes' Aggregate Model

The adjustment functions (**24**) - (**35**) define the way in which changes in twelve endogenous variables are determined in Keynes' aggregate model: R_t, M_t^w, P_t^a, P_t^i, P_t^c, C_t^w, C_t^{we}, I_t^w, I_t^{we}, Y_t^{ew}, N_t^w, and Y_t^w. Since these functions are assumed to pass through the origin the system is in equilibrium in the sense that there is no reason for any variable to change when all of the adjustment functions are equal to zero. This gives us twelve equilibrium conditions which contain twelve endogenous variables as summarized in **Table 1**. This table outlines the mathematical structure of the short-run aggregate model described by Keynes in the passages quoted above. The equilibrium values of the endogenous variables are assumed to be determined by the behavioral relationships defined by the aggregate behavioral equations (**1**) - (**23**) *given the assumption that employment is determined by the effective demands of producers as their*

[9] Cf., Keynes: "the effects on employment of the realized sale-proceeds of recent output and those of the sale-proceeds expected from current input; and producers' forecasts are more often gradually modified in the light of results than in anticipation of prospective changes" (1936, p. 51).

*expectations adjust to equate their effective demands to the actual demands that exist in markets by way of the adjustment functions (**24**) - (**35**).*

Table 1: Structure of Keynes' Aggregate Model		
Market	**Equilibrium Conditions**	**Endogenous Variables**
Assets	$M_t^{ws}= M_t^w$; $M_t^{ws}= M_t^{wd}$ $A_t^{wd}=A_t^w$	M_t^w, R_t, P_t^a
Investment	$I_t^{we}= I_t^{wd}$; $I_t^{we}=I_t^w$ $P_t^{id} = P_t^{is}$	I_t^{we}, I_t^w, P_t^i
Consumption	$C_t^{we}= C_t^{wd}$; $C_t^{we}=C_t^w$ $P_t^{cd} = P_t^{cs}$	C_t^{we}, C_t^w, P_t^c
Labor	$Y_t^{ws}=Y_t^{we}$	N_t^w
Identities	$Y_t^{we} = C_t^{we} + I_t^{we}$ $Y_t^w = C_t^w + I_t^w$	Y_t^{we}, Y_t^w

3. Achieving Equilibrium

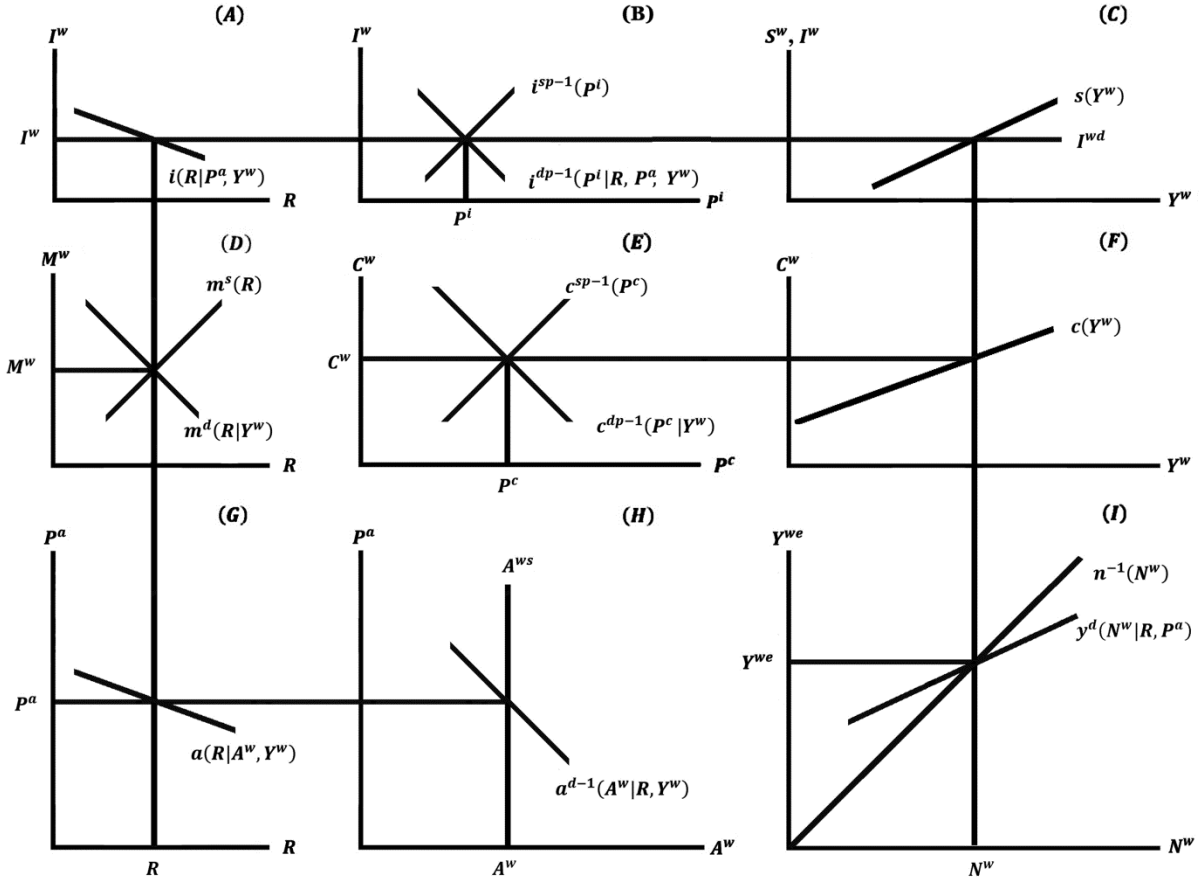
The way in which the short-run equilibrium values of the variables in the model are determined by suppliers and demanders in individual markets and the ways in which the interconnections between markets are summarized by the non-debt asset equilibrium function (**7**) and Keynes' consumption function (**11**) and MEC schedule (**16**) are illustrated in **Figure 1** where: [10]

1. Given the rate of interest $\boldsymbol{R}$, the price of non-debt assets $\boldsymbol{P^a}$, and output/income $\boldsymbol{Y^w}$ the price $\boldsymbol{P^i}$ and rate of investment goods production $\boldsymbol{I^w}$ are determined in panel (**B**) by the demanders and suppliers of investment goods as dictated by the inverses of the demand price $\boldsymbol{i^{dp-1}(P^i\,|R, P^a, Y_t^w)}$ (**14**) and supply price $\boldsymbol{i^{sp-1}(P^i)}$ (**13**) of investment goods functions from which Keynes' MEC schedule $\boldsymbol{i(R|P^a, Y^w)}$ (**16**) in panel (**A**) is derived.
2. Given the rate of investment demand $\boldsymbol{I^{wd}}$ the value of output/income $\boldsymbol{Y^w}$ is determined by savers and investors in accordance with Keynes' savings function $\boldsymbol{s(Y^w)}$ (**12**) in panel (**C**) which is given by one minus Keynes' consumption function $\boldsymbol{c(Y^w)}$ (**11**) in panel (**F**).
3. Given output/income $\boldsymbol{Y^w}$ the rate of interest $\boldsymbol{R}$ is determine by the demanders and suppliers of money as dictated by the demand $\boldsymbol{m^d(R|Y^w)}$ (**2**) and supply $\boldsymbol{m^s(R)}$ (**3**) of money functions in panel (**D**), and the price $\boldsymbol{P^c}$ and rate of consumption goods production $\boldsymbol{C^w}$ are determined in panel (**E**) by the demanders and suppliers of consumption goods as dictated by the inverses of the demand price $\boldsymbol{c^{dp-1}(P^c\,|Y^w)}$ (**9**) and supply price $\boldsymbol{c^{sp-1}(P^c)}$ (**8**) of consumption goods functions from which Keynes' consumption function $\boldsymbol{c(Y^w)}$ (**11**) in panel (**F**) is derived.
4. Given the rate of interest $\boldsymbol{R}$, stock of non-debt assets $\boldsymbol{A^w}$, and level of output/income $\boldsymbol{Y^w}$ the

[10] For a detailed discussion of the way in which equilibrium is defined and achieved in the works of Marshall, Keynes, and neoclassical economists see Hayes (2006), Kregel, and Lavoie and Godley. See also Blackford (2021).

price of non-debt assets $\boldsymbol{P^a}$ is determine in panel (**H**) by the demanders and suppliers of non-debt assets as dictated by the supply of non-debt assets $\boldsymbol{A^{ws}}$ (**4**) and the inverse of the demand for non-debt assets $\boldsymbol{a^{d-1}(A^w|R,Y^w)}$ (**5**) functions from which the non-debt asset equilibrium function $\boldsymbol{a(R|A^w,Y^w)}$ (**7**) in panel (**G**) is derived.

Figure 1: Short-Run Equilibrium.



5. Given the rate of interest $\boldsymbol{R}$ and the price of non-debt assets $\boldsymbol{P_t^a}$ the point of effective demand $\boldsymbol{Y^{we}}$ and rate of employment $\boldsymbol{N^w}$ as determined by producers in the consumption and investment goods industries is given at the intersection of Keynes' aggregate supply $\boldsymbol{n^{-1}(N^w)}$ (**22**) and demand $\boldsymbol{y^d(N^w|R,P^a)}$ (**23**) schedules in panel (**I**) as output/income $\boldsymbol{Y^w}$ adjusts to equate saving $\boldsymbol{S^w}$ and investment $\boldsymbol{I^w}$ in panel (**C**).

But what is most significant about the model embodied in equations (**1**) through (**35**) above is that it formalizes the analytical framework develop by Keynes throughout *The General Theory*—a framework within which a *logically consistent, causal analysis of* the *dynamic behavior of the economic system is possible.*

3.1 Changes in Short-Run Equilibrium

As noted above, rather than view the economic system from the perspective of a set of Walrasian equations Keynes viewed the system from the perspective of a set of Marshallian partial equilibrium models in which *the values of individual variables are determined by the choices of those decision-making units that actually have the power to determine the value of*

each variable at each point in time as the system evolves through time. Accordingly, in examining the dynamic behavior of the model specified above it is assumed that given the money wage $\boldsymbol{W_t}$, stock of non-debt assets $\boldsymbol{A_t^w}$, and the other exogenous variables and parameters of the model:

1. The prices and rates of production and sale of goods and resources along with the price of non-debt assets (i.e., the complex of prices on real and financial non-debt assets) is (are) determined through the interactions of suppliers and demanders in the markets for goods, resources, and non-debt assets.
2. The rate of interest (i.e., the complex of rates of interest on new loans and debt assets) is (are) determined by the suppliers and demanders for money (i.e., liquidity) in the money market.
3. Employment is determined by producers in accordance with their *effective demands*—that is, at the point at which producers *expect* to maximize their profits.
4. Income (i.e., the value of output produced) is determined by savers and investors as they interact in the markets for consumption and investment goods.
5. And the entire process by which these variables are determined *at each point in time* is governed by the *expectations* of decision-making units as their expectations adjust to the realized results that are achieved within the system as the system evolves *through time*.

These assumptions make it possible to isolate those factors that *directly* and *in themselves* determine each variable at *each point in time* whether the *system* is in equilibrium or not. This, in turn, makes it possible to establish *the temporal order in which events must occur* as decision-making units respond to changes in the exogenous determinants of the variables *in each sector* of the economy. It is the ability to establish the temporal order in which events must occur within the analytical framework developed by Keynes throughout *The General Theory* that makes a *logically consistent, causal analysis of the dynamic behavior* possible in Keynes' general theory and in economics in general.[11]

Consider, for example, an increase in thriftiness that takes the form of an increase in the demand for securities—that is, a decrease in the demand for consumption goods (**9**) that takes the form of an increase in the demand for non-debt assets (**5**). How will this affect the short-run equilibrium position of the economic system shown in **Figure 1**, and *how will the new short-run equilibrium come about?*

The *direct* effects will be to 1) *decrease* the demand for consumption goods $\boldsymbol{c^{dp-1}(P^c|Y^w)}$ (**9**) in panel (**E**) of **Figure 1** which will decrease the consumption function $\boldsymbol{c(Y^w)}$ (**11**) in panel (**F**) and increase the saving function $\boldsymbol{s(Y^w)}$ (**12**) in panel (**C**), and 2) to *increase* the demand for non-debt assets $\boldsymbol{a^{d-1}(A^w|R,Y^w)}$ (**5**) in panel (**H**). Thus, the direct effects will be to create an excess of savings over investment in panel (**C**), an excess supply in the market for consumption goods in panel (**E**), and an excess demand for non-debt assets in panel (**H**).

The excess demand for non-debt assets in panel (**H**) will put pressure on the price of non-debt assets $\boldsymbol{P^a}$ to increase in accordance with (**26**), and as producers in the consumption goods industries respond to the excess supply for consumption goods and adjust their effective demands for consumption goods $\boldsymbol{C^{we}}$ to the actual demands for consumption goods $\boldsymbol{C^{wd}}$ in

[11] See Blackford (2022) and Hume, and cf., Brady, Hayes, and Lavoie and Godley.

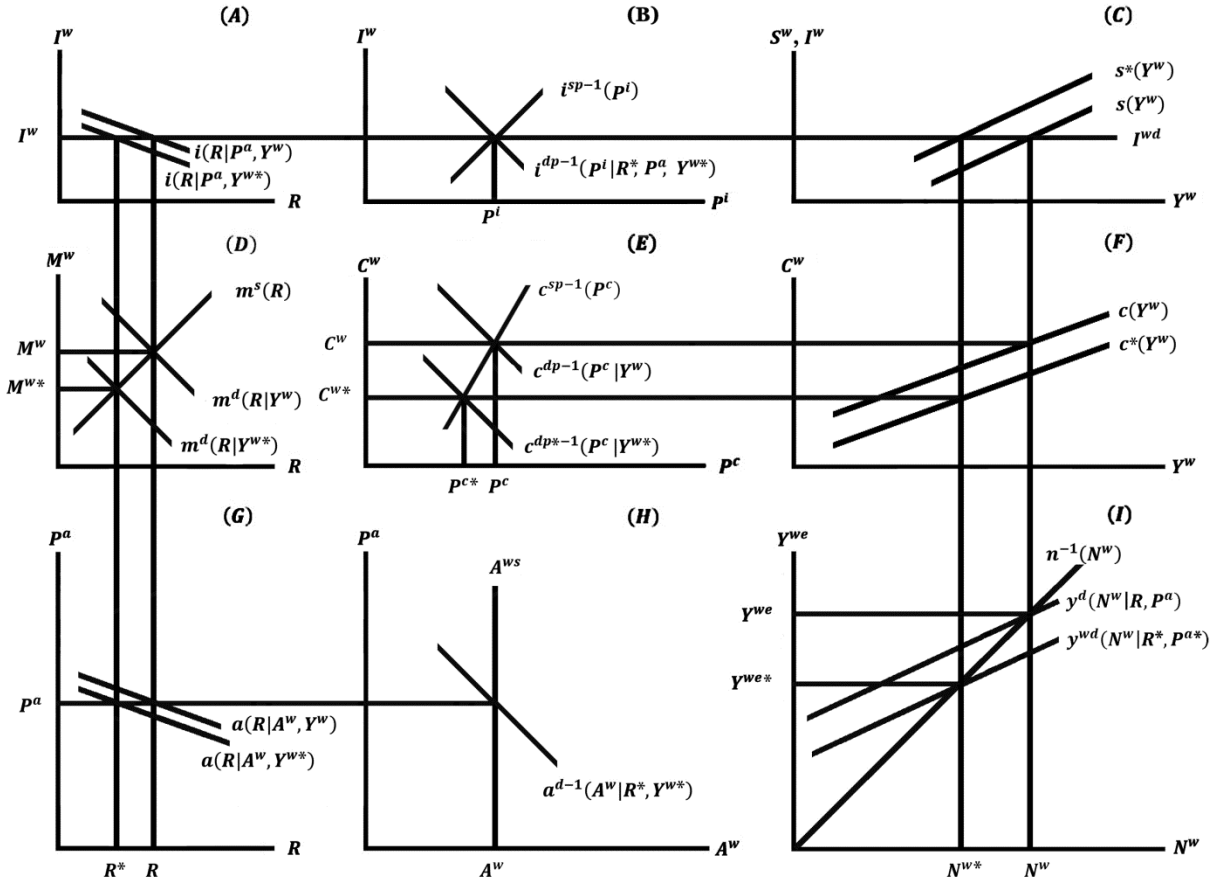
accordance with (**27**), the price $\boldsymbol{P^c}$ and production $\boldsymbol{C^w}$ of consumption goods in panel (**E**) will begin to fall in accordance with (**32**) and (**29**) with a concomitant fall in aggregate demand $\boldsymbol{y^d(N^w|R,P^a)}$ (**23**) in panel (**I**) as the effective demand for output $\boldsymbol{Y^{we}}$ adjusts to the actual demand for output $\boldsymbol{Y^{wd}}$ (**23**) in accordance with (**34**). This will lead to a fall in employment $\boldsymbol{N^w}$ and output/income $\boldsymbol{Y^w}$ in accordance with (**33**) and (**35**).

The resulting fall in output/income $\boldsymbol{Y^w}$ will, in turn, have negative effects on the demands for money $\boldsymbol{m^d(R|Y^w)}$ (**2**) in panel (**D**), investment goods $\boldsymbol{i^{dp-1}(P^i|R,P^a,Y^w)}$ (**14**) in panel (**B**), consumption $\boldsymbol{c^{dp-1}(P^c|Y^w)}$ (**9**) goods in panel (**E**), and non-debt assets $\boldsymbol{a^{d-1}(A^w|R,Y^w)}$ (**5**) in panel (**G**). The effects on the demand for non-debt assets $\boldsymbol{a^{d-1}(A^w|R,Y^w)}$ (**5**) in panel (**H**) will put pressure on the price of non-debt assets $\boldsymbol{P^a}$ to fall in accordance with (**26**), and the resulting excess demand for money in panel (**D**) will cause the rate of interest $\boldsymbol{R}$ and stock of money in existence $\boldsymbol{M^w}$ to fall in accordance with (**24**) and (**25**), respectively, which, in turn, will have positive effects on the demands for investment goods $\boldsymbol{i^{dp-1}(P^i|R,P^a,Y_t^w)}$ (**14**) in panel (**B**), non-debt assets $\boldsymbol{a^{d-1}(A^w|R,Y^w)}$ (**5**) in panel (**G**), and aggregate demand $\boldsymbol{y^d(N^w|R,P^a)}$ (**23**) in panel (**I**). At the same time, the decrease in the value of aggregate output/income $\boldsymbol{Y^w}$ that *results* from the decrease in the output of consumption goods $\boldsymbol{C^w}$ will have the effect of further decreasing the demand for consumption goods $\boldsymbol{c^{dp-1}(P^c|Y^w)}$ (**11**) in panel (**E**). This, in turn, will set in motion a *causal* feedback loop within the system (the multiplier).

As the effective demand for consumption goods $\boldsymbol{C^{we}}$ adjusts to the actual demand for consumption goods $\boldsymbol{C^{wd}}$ in accordance with (**27**) producers will decrease the output of consumption goods $\boldsymbol{C^w}$ in accordance with (**29**) as the price $\boldsymbol{P^c}$ of consumptions goods falls in accordance with (**32**). This will further decrease the aggregate demand $\boldsymbol{y^d(N^w|R,P^a)}$ (**23**) in panel (**I**) which will *cause* a further decrease aggregate effective demand $\boldsymbol{Y^{we}}$ (**21**), aggregate employment $\boldsymbol{N^w}$, and output/income $\boldsymbol{Y^w}$ in accordance with (**34**), (**33**), and (**35**), respectively, which will further decrease the demands for consumption goods $\boldsymbol{c^{dp-1}(P^c|Y^w)}$ (**9**) in panel (**E**) and money $\boldsymbol{m^d(R|Y^w)}$ (**2**) in panel (**D**). The further decrease in aggregate output/income $\boldsymbol{Y^w}$ *caused* by the further decrease in consumption goods produced $\boldsymbol{C^w}$ will *cause* a further decrease in the price $\boldsymbol{P^c}$ and output $\boldsymbol{C^w}$ of consumption goods which will *cause* a further decrease in employment $\boldsymbol{N^w}$, output/income $\boldsymbol{Y^w}$, and the demand for money $\boldsymbol{m^d(R|Y^w)}$ (**2**) which will *cause* a further decrease in the rate of interest $\boldsymbol{R}$ and stock of money in existence $\boldsymbol{M^w}$. The fall in output/income $\boldsymbol{Y^w}$ will enhance the negative effects on the demands for investment goods $\boldsymbol{i^{dp-1}(P^i|R,P^a,Y_t^w)}$ (**14**) and non-debt assets $\boldsymbol{a^{d-1}(A^w|R,Y^w)}$ (**5**) while the resulting fall in the rate of interest $\boldsymbol{R}$ will further enhance the positive effects on these demands. This causal loop must continue until system has achieved the short-run, system equilibrium depicted in **Figure 2** where the rate of interest, consumption, employment, price of consumption goods, and output/income have decreased from $\boldsymbol{R}$, $\boldsymbol{M^w}$, $\boldsymbol{N^w}$, $\boldsymbol{P^C}$, $\boldsymbol{C^w}$, and $\boldsymbol{Y^w}$ to $\boldsymbol{R^*}$, $\boldsymbol{M^{w*}}$, $\boldsymbol{N^{w*}}$, $\boldsymbol{P^{C*}}$, $\boldsymbol{C^{w*}}$, and $\boldsymbol{Y^{w*}}$.

The prices of non-debt assets $\boldsymbol{P^a}$ and investment goods $\boldsymbol{P^i}$ along with the level of investment $\boldsymbol{I^w}$ are left unchanged in this figure since the direction of change of these variables is indeterminate; whether these variables increase or decrease depends on the relative strengths of the positive effects of the fall in the rate of interest $\boldsymbol{R}$ on the demand for investment goods $\boldsymbol{i^{dp-1}(P^i|R,P^a,Y_t^w)}$ (**14**) in panel (**B**) and the demand for non-debt assets $\boldsymbol{a^d(P^a|R,Y_t^w)}$ (**5**) in panel (**H**) and the negative effects of the fall in output/income $\boldsymbol{Y^w}$ on these two demands.

Figure 2: An Increase in Thriftiness.



3.2 Achieving *Long-Run Equilibrium*

The stock of non-debt assets $\boldsymbol{A_t^w}$ is assumed to be exogenous in Keynes' short-run model as specified above, but positive net investment must, by definition, increase $\boldsymbol{A_t^w}$ over time. While it is reasonable to ignore the effects of this increase in the short run, Keynes argued that these effects can be dramatic in the long run. Specifically, he argued that the "position of [long-run] equilibrium, under conditions of *laissez-faire*, will be one in which employment is low enough and the standard of life sufficiently miserable to bring savings to zero." (1936, p. 217-18)

The nature of the long-run equilibrium envisioned by Keynes can be demonstrated in the model specified above by noting that "an increased investment in any given type of capital during any period of time the marginal efficiency of that type of capital will diminish as the investment in it is increased" (Keynes, 1936, p. 136) and explicitly including the stock of non-debt assets $\boldsymbol{A_t^w}$ in the demand price of investment function (**14**) to obtain:

$$\boldsymbol{P_t^{id} = i^{dp}(I_t^w, R_t, P_t^a, Y_t^w, A_t^w),\quad i_3^{dp},\ i_4^{dp} > 0,\quad i_1^{dp},\ i_2^{dp},\ i_5^{dp} < 0} \qquad (\boldsymbol{14a})$$

This implies that the MEC (**16**) and aggregate demand (**23**) functions become:

$$\boldsymbol{I_t^{wd} = i(R_t, P_t^a, Y_t^w, A_t^w),\quad i_2, i_3 > 0,\quad i_1, i_4 < 0} \qquad (\boldsymbol{16a})$$

$$Y_t^{wd} = c\left(n^{-1}(N_t^w)\right) + i(R_t, P_t^a, n^{-1}(N_t^w), A_t^w) \qquad (23a)$$
$$= y^d(N_t^w, R_t, P_t^a, A_t^w), \;\; y_1^d, y_3^d > 0, \quad y_2^d, y_4^d < 0$$

and if, for ease of exposition, we assume constant returns to scale and a fixed money wage W_t the consumption (**8**) and investment (**13**) goods supply-price functions can be replaced with their long-run counterparts:

$$P_t^{cs} = P^{c*} \qquad (8a)$$

$$P_t^{is} = P^{i*} \qquad (13a)$$

Finally, the adjustment equations (**28**) and (**34**) must be modified accordingly:

$$dI_t^{we} = g^{ie}\left(I_t^{wd} - I_t^{we}\right) \qquad (28a)$$
$$= g^{ie}\left(i^{dp-1}\left(P_t^{i*}, R_t, P_t^a, Y_t^w, A_t^w\right) - I_t^{we}\right)$$
$$dY_t^{we} = dC_t^{we} + dI_t^{we} \qquad (34a)$$
$$= g^{ce}\left(c^{dp-1}(P_t^c, Y_t^w) - C_t^{we}\right) + g^{ie}\left(i^{dp-1}\left(P_t^i, R_t, P_t^a, Y_t^w, A_t^w\right) - I_t^{we}\right)$$

Given these extensions of the model the mechanisms by which an increase in the capital stock leads to Keynes' long-run equilibrium can be demonstrated by examining the effects of an increase in the stock of non-debt assets from A^w to A^{w*} in panel (**H**) of **Figure 3**. The direct effects of this increase in A^w will be to increase the supply of non-debt assets A^{ws} (**4**) in panel (**H**) and to decrease the demand for investment goods $i^{dp-1}(P^i|R, P^a, Y^w, A^w)$ (**14a**) in panel (**B**) and aggregate demand $y^{wd}(N^w|R, P^a, A^w)$ (**23a**) in panel (**I**). The result will be excess supplies of non-debt assets in panel (**H**), investment goods in panel (**B**), and aggregate demand in panel (**I**).

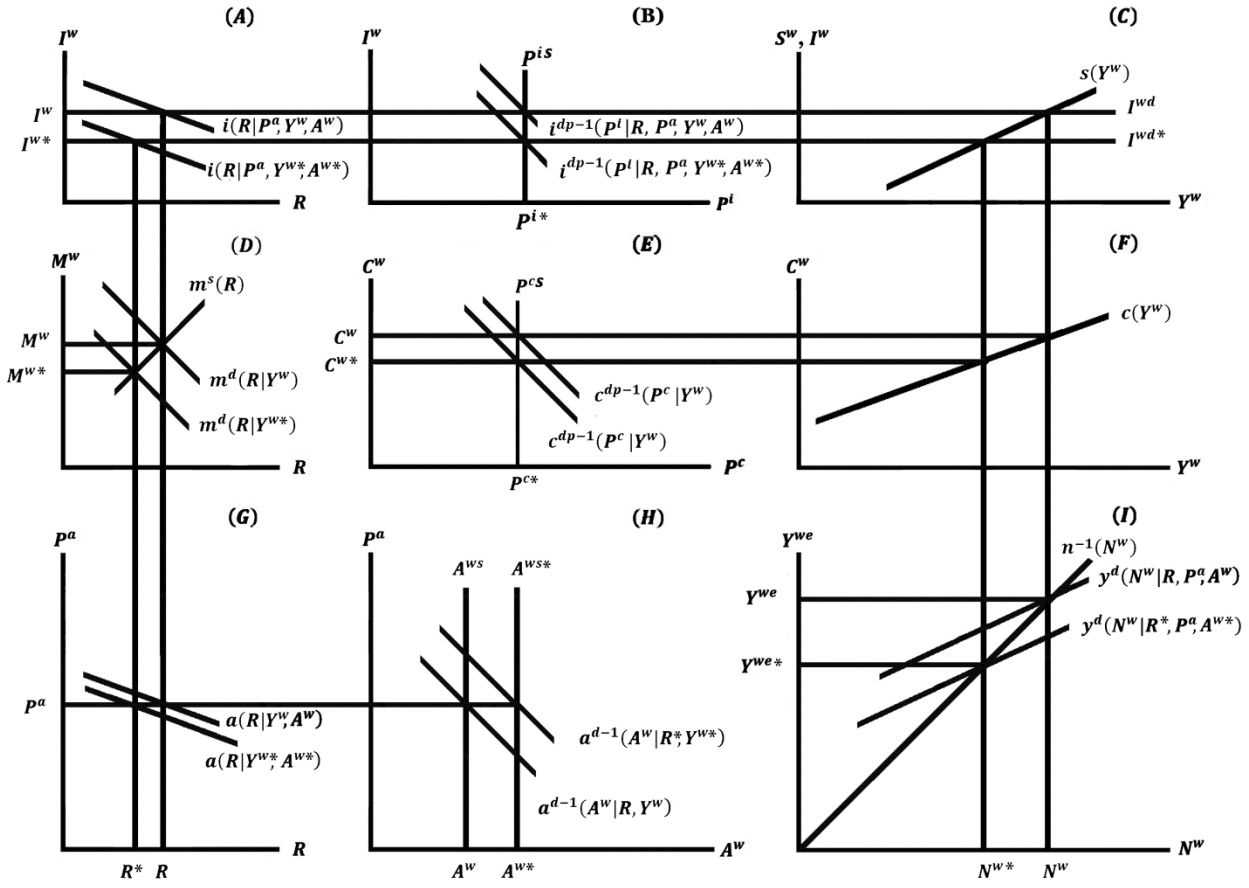
The excess supply of non-debt assets will cause the price of non-debt assets P^a in panel (**G**) to fall in accordance with (**26**) which will cause a further reduction in the demand for investment goods $i^{dp-1}(P^i|R, P^a, Y^w, A^w)$ (**14a**) in panel (**B**). As the effective demand for investment goods I^{we} adjusts to the actual demand for investment goods I^{wd} in accordance with (**28a**) the output of investment goods I^w will fall in accordance with (**30**). At the same time, as the aggregate effective demand for output Y^{we} adjusts to the actual demand for output Y^{wd} in accordance with (**34a**) aggregate employment N^w and output/income Y^w will fall in accordance with (**33**) and (**35**).

The fall in output/income Y^w will, in turn, decrease the transactions demand for money which will cause a fall in the demand for money $m^{wd}(R|Y^w)$ (**5**) in panel (**D**). The resulting excess supply of money will cause a fall in the rate of interest R and stock of money demanded M^w (**2**) in accordance with (**24**) and (**25**), respectively, which will enhance the demand for non-debt assets $a^{d-1}(A^w|R, Y^w)$ (**5**) in panel (*H*), the demand for investment goods $i^{dp-1}(P^i|R, P^a, Y^w, A^w)$ (**14a**) in panel (**B**), and the aggregate demand curve $y^{wd}(N^w|R, P^a, A^w)$ (**23a**) in panel (**I**). At the same time, the fall in output/income Y^w will also cause a fall in the demand for consumption goods $c^{wd-1}(P^c|Y^w)$ (**9**) in panel (**E**).

As the effective demand for consumption goods C^{we} adjusts to the actual demand for consumption goods C^{wd} in accordance with (**27**) the output of consumption goods C^w will fall in

accordance with (**29**). This, in turn, will cause a further fall in output and income $\boldsymbol{Y^w}$ as the multiplier process moves the system to its new point of short-run equilibrium at $\boldsymbol{A^{w*}}$, $\boldsymbol{P^a}$, $\boldsymbol{C^{w*}}$, $\boldsymbol{P^{c*}}$, $\boldsymbol{I^{w*}}$, $\boldsymbol{P^{i*}}$, $\boldsymbol{N^{w*}}$, $\boldsymbol{Y^{w*}}$, $\boldsymbol{M^{w*}}$, and $\boldsymbol{R^*}$ in **Figure 3**.

Figure 3: Toward Long-Run Equilibrium.



The price of non-debt assets $\boldsymbol{P^a}$ is left unchanged in this figure since whether this variable increases or decreases depends on the relative strengths of the negative effects of the fall in output/income $\boldsymbol{Y^w}$ and the positive effects of the fall in the rate of interest $\boldsymbol{R}$ on the demand for non-debt assets $\boldsymbol{a^{d-1}(A^w|R, Y^w)}$ (**5**) in panel (**H**) and, hence, is indeterminate.[12]

As the stock of non-debt assets $\boldsymbol{A^w}$ continues to increase over time, the levels of investment $\boldsymbol{I^w}$, consumption $\boldsymbol{C^w}$, employment $\boldsymbol{N^w}$, output/income $\boldsymbol{Y^w}$, and the rate of interest $\boldsymbol{R}$ must

[12] It should be noted that even though the positive effects of the fall in the rate of interest $\boldsymbol{R}$ on the demand for investment goods $\boldsymbol{i^{dp-1}(P^i|R, P^a, Y^w, A^w)}$ (**14a**) in panel (**B**) inhibit the negative effects of the increase in non-debt assets $\boldsymbol{A^w}$ it is not assumed that level of investment $\boldsymbol{I_t^w}$ is indeterminate since the rate of interest $\boldsymbol{R}$ can only fall in this situation *as a result* of the fall in output/income $\boldsymbol{Y^w}$ that is *caused* by a fall in investment $\boldsymbol{I^w}$; a fall in the rate of interest $\boldsymbol{R}$ cannot offset the fall in investment $\boldsymbol{I^w}$ that *caused* the fall in output/income $\boldsymbol{Y^w}$ that *caused* the fall in the rate of interest $\boldsymbol{R}$. Nothing in the system can change except the stock of non-debt assets $\boldsymbol{A^w}$ if investment does not fall in this situation.

continue to fall until the investment goods demand function $\boldsymbol{i^{dp-1}(P^i|R, P^a, Y^w, A^w)}$ (**14a**) in panel (**B**) has fallen to the point at which investment $\boldsymbol{I^w}$ is just sufficient to replace the capital that is consumed in the process of producing $\boldsymbol{Y^w}$. At that point the long-run equilibrium will have been achieved as *net* investment is forced to zero and the stock of non-debt assets $\boldsymbol{A^w}$ no longer increases.[13]

4. Changes in Exogenous Variables and Parameters

The fact that exogenous variables and parameters are assumed to be given in the analysis above does not mean that these factors cannot or will not change. It only means that the effects of changes in these factors can either be ignored in considering the problem at hand (such as the stock on non-debt assets $\boldsymbol{A^w}$ in short-run analysis) or that they are not determined in a systematic way within the system such that their effects must be examined separately. The effects of changes in two of these factors in the model specified above are particularly relevant in the history of economic controversy.

4.1 Changes in the Supply of Money

The first is the effects of an increase in the supply of money $\boldsymbol{m^s(R)}$ with regard to the short-run equilibrium examined in **Figure 2** and long-run equilibrium examined in **Figure 4**. Dennis H. Robertson, for example, argued that **Figure 2** exemplifies "a situation calling for a progressive increase in the supply of money" (1936, p. 180). This can be represented in **Figure 2** by a shift in the money supply function $\boldsymbol{m^s}$(**R***) in panel (**D**) to the right which will create an excess supply of money that leads to a fall in the rate of interest $\boldsymbol{R}$ in accordance with (**24**). The resulting fall in $\boldsymbol{R}$ will increase the demand for non-debt assets $\boldsymbol{a^{d-1}*(A^w|R*, Y^w*)}$ in panel (**H**) and the demand for investment goods $\boldsymbol{i^{dp-1}*(P^i|R*, P^a*, Y^w*)}$ in panel (**B**). The resulting excess demands will cause an increase in a) the price of non-debt assets $\boldsymbol{P^a}$ in panel (**H**), b) the price $\boldsymbol{P}^{\mathrm{i}}$ and output $\boldsymbol{I^w}$ of investment goods in panel (**B**), and c) in employment $\boldsymbol{N^w}$ and output/income $\boldsymbol{Y^w}$ in panel (**I**) as the effective demands for investment goods $\boldsymbol{I^{we}}$ and output $\boldsymbol{Y^{we}}$ adjust to the actual demands for investment goods $\boldsymbol{I^{wd}}$ and output $\boldsymbol{Y^{wd}}$ in accordance with (**26**), (**30**), (**33**), (**35**), (**28**) and (**34**), respectively.

Investment $\boldsymbol{I^w}$, output/income $\boldsymbol{Y^w}$, and employment $\boldsymbol{N^w}$ can be expected to continue to

[13] See Keynes (1936, pp. 27-32, 136, 211-15, 217-8, 228-31) and Blackford (2021). These results illustrate Keynes' paradox of thrift and are contrary to those obtained in Lavoie and Godley (LG) (pp. 117, 363-4). The fundamental difference between the model specified above and that of LG is that the LG model assumes exogenous net investment is determined by "animal spirits" that remains positive even if the real rate of interest is nil and the size of the capital stock is assumed to have no affect on the investment goods demand function. (pp. 383-5) These assumptions are rather unrealistic. LG also assume that an increase in wealth has a positive effect on consumption which may or may not be unrealistic. Keynes does not deal with this phenomenon directly other than to suggest that the propensity to save increases rather than decreases with wealth (1936, pp. 31-2). To the extent that there are positive effects of an increase in wealth on consumption that more than offset the negative effects of an increasing capital stock on investment, employment and output/income can be expected to increase. This is an empirical issue that is beyond the scope of this paper. See Palley.

increase in this situation (assuming, of course, the existence of unemployed resources) only so long as *the effects of the fall in the rate of interest R more than offsets the effects of the increase in the stock of non-debt assets* $\boldsymbol{A^w}$ *on the demand for investment goods* $\boldsymbol{i^{dp-1}(P^i|R, P^a, Y^w, A^w)}$ (**14a**) *in* ***Figure 3***. To the extent the effects of the fall in the rate of interest $\boldsymbol{R}$ do not offset the effects of the increase in non-debt assets $\boldsymbol{A^w}$ on the demand for investment goods $\boldsymbol{i^{dp-1}(P^i|R, P^a, Y^w, A^w)}$ (**14a**) the system can be expected to approach the long-run equilibrium represented in **Figure 3** as the rate of interest $\boldsymbol{R}$ (after adjusting for the cost of bringing borrowers and lenders together) and net investment are forced to zero.[14]

4.2 Changes in the Money Wage

The second exogenous variable that is particularly relevant in the history of economic controversy is the effects of a decrease in the money wage $\boldsymbol{W}$ on the level of employment. Given the assumption that employment is determined by producers at the point of effective demand in Keynes' general theory the only way in which a change in the money wage can affect employment $\boldsymbol{N^w}$ is through an effect on the *expectations* of producers as these expectations adjust to the effects of a change in the money wage on a) the MEC, b) the propensity to consume, and c) the rate of interest. (1936, pp. 183-4) These three factors determine the *equilibrium* level of employment, and output/income in Keynes' model in that the *equilibrium* level of employment and output/income $\boldsymbol{Y_t^w}$ cannot change unless at least one of these factors change. This follows directly from Keynes' Marshallian aggregate supply (**22**) and demand (**23**) functions. Since:

1. the level of employment $\boldsymbol{N_t^w}$ is determined by effective demand $\boldsymbol{Y_t^{we}}$ in accordance with (**20**), and
2. effective demand is determined by the actual demand in accordance with (**34a**)

the *equilibrium* aggregate demand curve (**23**)—that is, *the aggregate demand curve for which expectations can be realized*—can change only by way of a change in a) the MEC, b) the propensity to consume, or c) the rate of interest. (Keynes 1936, chaps. 3, 20) This means that if, for example, a fall in the money wage were to increase expectations directly so as to induce producers to increase employment in the absence of a change in at least one of these three factors, all of the increased output that results can be sold "[o]nly if the community's marginal propensity to consume is equal to unity." (Keynes 1936, p. 261)[15] Barring this possibility, the

[14] Assuming, of course, that positive effects of an increase in wealth on consumption do not more than offset the negative effects of an increasing capital stock on investment. See footnote 1**3** above and Blackford (2021; 2022).

[15] Strictly speaking, (**11**), (**15**), (**22**) and (**23**) imply that the price of non-debt assets $\boldsymbol{P_t^a}$ must be given as well as the rate of interest $\boldsymbol{R_t}$ to require the MPC $\boldsymbol{c'}$ to be equal unity in this situation:

$$\boldsymbol{Y_t^{wd} = Y_t^{ws}}$$

$$\boldsymbol{c\left(n^{-1}(N_t^w)\right) + i(R_t, P_t^a) = n^{-1}(N_t^w)}$$

$$\boldsymbol{c'\mathrm{d}\left(n^{-1}(N_t^w)\right) + i_1 dR_t + i_2 dP_t^a = n^{-1'} dN_t^w}$$

$$\boldsymbol{c' n^{-1'} dN_t^w + i_1 dR_t + i_2 dP_t^a = n^{-1'} dN_t^w}$$

expectations of producers must eventually become reconciled to this reality, and any increase in employment that results solely from a *ceteris paribus* increase in expectations can only be temporary as inventories and debt accumulate and liquid assets are depleted.

Even though an increase in expectations that result from a fall in the money wage cannot *in itself* cause a change in the equilibrium level of employment and output in the absence of a change in one of the factors that determine the *position* of the aggregate demand function there are a number of ways in which a fall in the money wage can be expected to affect these three factors, and, thereby, change the equilibrium levels of employment and output/income. For example, to the extent a fall in the money wage lowers domestic wages relative to foreign wages it could lead to an increase net exports that leads to an increase in employment through an increase in the aggregate propensity to consume schedule $\boldsymbol{c}(\boldsymbol{Y^w})$ (**15**) in panel (**F**) of **Figure 1** by way of the decrease in foreign-sector saving. (Blackford 2021) The resulting increase in the aggregate demand schedule $\boldsymbol{y^d}(\boldsymbol{N^w}|\boldsymbol{R},\boldsymbol{P^a})$ (**23**) in panel (**I**) could, in turn, lead to a further increase in employment by increasing the investment demand curve $\boldsymbol{i^{dp-1}}(\boldsymbol{P^i}|\boldsymbol{R},\boldsymbol{P^a},\boldsymbol{Y^w})$ (**14**) in panel (**B**), provided, of course, this entire process is not cut short by an offsetting change in foreign exchange rates or relative prices.

A fall in the money wage may also have an effect on the demand for consumption goods $\boldsymbol{c^{wd-1}}(\boldsymbol{P^c}|\boldsymbol{Y^w})$ (**9**) in panel (**E**) and, thereby, the aggregate propensity to consume schedule $\boldsymbol{c}(\boldsymbol{Y^w})$ (**15**) in panel (**F**) and saving schedule $\boldsymbol{s}(\boldsymbol{Y^w})$ (**12**) in panel (**C**) through a redistribution of income from wage earners to the owners of other factors of production to the extent there are differences in the propensities to consume between these two groups, though this is apt to reduce the demand for consumption goods $\boldsymbol{c^{wd-1}}(\boldsymbol{P^c}|\boldsymbol{Y^w})$ (**9**) in panel (**E**) and propensity to consume schedule $\boldsymbol{c}(\boldsymbol{Y^w})$ (**15**) in panel (**F**) rather than increase them. And to the extent a fall in wages is accompanied by a fall in prices, the resulting fall in the transactions demand for money must cause the demand for money schedule $\boldsymbol{m^{wd}}(\boldsymbol{R}|\boldsymbol{Y^w})$ (**2**) in panel (**D**) to fall which given the supply of money $\boldsymbol{m^s}(\boldsymbol{R})$ (**3**) can be expected to reduce the rate of interest $\boldsymbol{R}$ causing an increase in the demand for investment goods $\boldsymbol{i^{dp-1}}(\boldsymbol{P^i}|\boldsymbol{R},\boldsymbol{P^a},\boldsymbol{Y^w})$ (**14**) in panel (**B**) provided the fall in wages and prices does not lead to an expectation of a further fall in wages and prices that has a negative effect on demand for investment $\boldsymbol{i^{dp-1}}(\boldsymbol{P^i}|\boldsymbol{R},\boldsymbol{P^a},\boldsymbol{Y^w})$ (**14**) in panel (**B**).

Keynes undertook a detailed examination of these and other aspects of the problem of explaining the effects of a fall in the money wage on employment in Chapter 19 of *The General Theory*, and argued that that a lack of rigidity in the money wage would be likely "to cause a great instability of prices, so violent perhaps as to make business calculations futile in an economic society functioning after the manner of that in which we live." (1936, p. 269) In the end he concluded:

> In the light of these considerations I am now of the opinion that the maintenance of a stable general level of money-wages is, on a balance of considerations, the most advisable policy for a closed system; whilst the same conclusion will hold good for an open system, provided that equilibrium with the rest of the world can be secured by

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$$\boldsymbol{c'}1\boldsymbol{dN_t^w} + 0 + \boldsymbol{i_2 dP_t^a} = \boldsymbol{1dN_t^w}$$

$$\boldsymbol{c'} + \boldsymbol{i_2}\frac{\boldsymbol{dP_t^a}}{\boldsymbol{dN_t^w}} = \boldsymbol{1}.$$

means of fluctuating exchanges. (1936, p. 270)

Thus, Keynes saw a lack of flexibility in the money wage, not as a cause of unemployment or economic stagnation but as a requisite condition for economic stability, and he saw no reason to believe that a falling money wage was a cure for unemployment in the short run or for economic stagnation in the long-run.

5. Mr. Keynes and the 'NeoClassics'

In one way or another the equilibrium conditions and behavioral equations of the aggregate model outlined in **Table 1** have been at the center of neoclassical macroeconomics since Keynes published *The General Theory of Employment, Interest, and Money* or at least since 1937 when Hicks published his iconic paper, "Mr. Keynes and the 'Classics': A Suggested Interpretation." (Patinkin, 1976, p. 1092) Disputes have arisen with regard to the choice of units, the choice of endogenous and exogenous variables and parameters, the way in which the endogenous variables are determined, the importance of and role played by expectations, the nature of the dynamic adjustment functions, the appropriate specification of the behavioral equations, and the nature of the micro-foundations of this model, but the general framework of the model is more or less as outlined above. There is, however, a fundamental difference between the way in which Keynes viewed this model and the neoclassical understanding of it as exemplified by the analytical framework within which Hicks chose to explain his interpretation of Keynes and the classics.

5.1 Hicks' Two-Good Model

Hicks began his interpretation by assuming the existence of two short-run production functions:

$$\boldsymbol{C = f^c(N^c),} \qquad \boldsymbol{f^{c'} > 0,} \quad \boldsymbol{f^{c''} < 0} \tag{36}$$

$$\boldsymbol{I = f^i(N^i),} \qquad \boldsymbol{f^{i'} > 0,} \quad \boldsymbol{f^{i''} < 0} \tag{37}$$

where $\boldsymbol{C}$ and $\boldsymbol{I}$ denote the output of consumption and investment goods, and $\boldsymbol{N^C}$ and $\boldsymbol{N^i}$ denote the input of labor devoted to the production of each of these goods. He also assumed that the prices of consumption ($\boldsymbol{P^c}$) and of investment goods ($\boldsymbol{P^i}$) are equal to their marginal costs:

$$\boldsymbol{P^c = W/f^{c'}(N^c),} \qquad \boldsymbol{\partial P^c/\partial N^c > 0} \tag{38}$$

$$\boldsymbol{P^i = W/f^{i'}(N^i),} \qquad \boldsymbol{\partial P^i/\partial N^i > 0} \tag{39}$$

Hicks then argued that income earned in the consumption goods sector ($\boldsymbol{Y^c}$), income earned in the investment goods sector ($\boldsymbol{Y^i}$), and total income ($\boldsymbol{Y}$) can be written as:

$$\boldsymbol{Y^c = P^c f^c(N^c)} \tag{40}$$

$$\boldsymbol{= W f^c(N^c)/f^{c'}(N^c),} \qquad \boldsymbol{\partial Y^c/\partial N^c > 0}$$

$$\boldsymbol{Y^i = P^i f^i(N^i)} \tag{41}$$

$$\boldsymbol{= W f^i(N^i)/f^{i'}(N^i),} \qquad \boldsymbol{\partial Y^i/\partial N^i > 0}$$

$$\boldsymbol{Y = P^c f^c(N^c) + P^i f^i(N^i)} \tag{42}$$

$$\boldsymbol{= \frac{W f^c(N^c)}{f^{c'}(N^c)} + \frac{W f^i(N^i)}{f^{i'}(N^i)}}$$

and concluded that if the stock of capital and the money wage ($\boldsymbol{W}$) are exogenously determined "once [$\boldsymbol{Y}$] and [$\boldsymbol{Y^i}$] are determined, [$\boldsymbol{N^c}$] and [$\boldsymbol{N^c}$] can be determined." (p. 148)

Having structured the problem of explaining the level of employment (i.e., the sum of $\boldsymbol{N^c}$ and $\boldsymbol{N^i}$) in this way Hicks offered his interpretation of the classical solution to this problem by adding the "Cambridge Quantity equation" which assumes a proportional relationship ($\boldsymbol{k}$) between the exogenously determined stock of money ($\boldsymbol{M}$) and total income $\boldsymbol{Y}$:

$$\boldsymbol{M = kY} \qquad (\mathbf{43})$$

and concluded: "As soon as $\boldsymbol{k}$ is given, total Income is therefore determined."[16] (p. 149) He then argued that income earned in the production of investment goods $\boldsymbol{Y^i}$ (=$\boldsymbol{P^iI}$) is a function of the rate of the rate of interest:

$$\boldsymbol{Y^i = i(R)}, \qquad \boldsymbol{i' < 0} \qquad (\mathbf{44})$$

and stated: "This is what becomes the marginal-efficiency-of-capital schedule in Mr. Keynes' work." Next, Hicks added the saving/investment equilibrium condition:

$$\boldsymbol{Y^i = s(Y, R)}, \quad \boldsymbol{s_1 > 0}, \quad \boldsymbol{s_2 < 0} \qquad (\mathbf{45})$$

where savings is assumed to be a function $\boldsymbol{s}$ of both total income $\boldsymbol{Y}$ and the rate of interest $\boldsymbol{R}$. After noting that: "Since Income is already determined, we do not need to bother about inserting Income here unless we choose," he argued that taking (**43**) - (**44**) "as a system … we have three fundamental equations … to determine three unknowns [$\boldsymbol{Y}$, $\boldsymbol{Y^i}$, $\boldsymbol{R}$]…. [$\boldsymbol{N^i}$] and [$\boldsymbol{N^c}$] can be determined from [$\boldsymbol{Y}$] and [$\boldsymbol{Y^i}$]. Total employment, [$\boldsymbol{N^i}$] + [$\boldsymbol{N^c}$], is therefore determined." (p. 149)[17]

Having outlined his interpretation of the classical solution to the problem of explaining the level of employment in this way Hicks then offered his interpretation of Keynes' solution by replacing the Cambridge equation (**43**) in the classical model with Keynes' liquidity preference equilibrium condition:

$$\boldsymbol{M = m^d(Y, R)}, \quad \boldsymbol{m_1^d > 0}, \ \boldsymbol{m_2^d < 0}. \qquad (\mathbf{46})$$

He then argued that income earned in the production of investment goods $\boldsymbol{Y^i}$ should be a function of both the rate of interest $\boldsymbol{R}$ and total income $\boldsymbol{Y}$ and replaced his classical version of Keynes' MEC schedule (**44**) with:

$$\boldsymbol{Y^i = i^*(Y, R)}, \quad \boldsymbol{i_1^* > 0}, \ \boldsymbol{i_2^* < 0}. \qquad (\mathbf{47})$$

Equations (**45**) - (**47**) are central to Hicks' analytical framework in that (**46**) defines Hicks' LM schedule (those combinations of $\boldsymbol{Y}$ and $\boldsymbol{R}$ for which the supply and demand for money are

[16] This is decidedly *not* the way in which variables are 'determined' in Keynes' general theory. Keynes viewed the determination of variables as an economic problem, not as a mathematical problem. Variables are determined by the actions of those decision-making units that actually have the *power* to determine variables as they interact in markets in Keynes' general theory, not by counting mathematical equations. Equation counting is a necessary, but not a sufficient condition for an explanation of economic behavior to be meaningful in Keynes' general theory or in economics in general. See Keynes (1936, pp. 23-35, 46-7, 245-55, 257-71, 280-91, 297-8).

[17] Again, this is decidedly *not* the way variables are 'determined' in Keynes' general theory.

equal) and (**45**) and (**47**) can be combined to obtain Hicks' IS schedule (those combinations of $\boldsymbol{Y}$ and $\boldsymbol{R}$ for which desired saving and investment are equal):

$$\boldsymbol{i^*(Y,R) = s(Y,R)}. \qquad (\mathbf{48})$$

Hicks then used these two schedules to solve for the rate of interest $\boldsymbol{R}$ and income $\boldsymbol{Y}$, the values of which presumably make it possible to solve (**36**) – (**42**) for $\boldsymbol{Y^i}$, $\boldsymbol{C}$, $\boldsymbol{I}$, $\boldsymbol{P^c}$, $\boldsymbol{P^i}$, $\boldsymbol{N^c}$ and $\boldsymbol{N^i}$ in terms of the exogenously determined money wage $\boldsymbol{W}$ and stock of money $\boldsymbol{M}$.

5.2 Keynesian One-Good Model

Hicks' model became the backbone of Keynesian economics in the 1950s in the name of what Samuelson called the Neoclassical Synthesis. (Weintraub; Zouache) In the process, Hicks' two-good model was converted to a one-good model by a) replacing the individual prices $\boldsymbol{P^i}$ and $\boldsymbol{P^c}$ with a single price ($\boldsymbol{P}$), b) replacing the individual levels of employment $\boldsymbol{N^i}$ and $\boldsymbol{N^c}$ by a single level of employment ($\boldsymbol{N}$), and c) replacing equations (**36**) - (**42**) with their aggregate one-good counterparts:

$$\boldsymbol{Q = f(N), \quad f' > 0, \ f'' < 0} \qquad (\mathbf{49})$$

$$\boldsymbol{Y = Pf(N)} \qquad (\mathbf{51})$$

$$\boldsymbol{= Wf(N)/f'(N), \quad \partial Y/\partial N > 0}$$

where $\boldsymbol{Q}$ is the aggregate output produced ($\boldsymbol{C + I}$), and $\boldsymbol{f}$ is an aggregate production function.

These modifications greatly simplified the way in which the short-run, static-equilibrium level of employment $\boldsymbol{N}$ could be explained in terms of the exogenously determined money wage $\boldsymbol{W}$ and stock of money $\boldsymbol{M}$ by reducing the number of variables to be solved for from nine ($\boldsymbol{Y}$, $\boldsymbol{R}$, $\boldsymbol{Y^i}$, $\boldsymbol{C}$, $\boldsymbol{I}$, $\boldsymbol{P^c}$, $\boldsymbol{P^i}$, $\boldsymbol{N^c}$, $\boldsymbol{N^i}$) to six ($\boldsymbol{Q}$, $\boldsymbol{Y}$, $\boldsymbol{R}$, $\boldsymbol{Y^i}$, $\boldsymbol{P}$, $\boldsymbol{N}$) which reduced the requisite number of equations to six (**46**) – (**51**) as well. Once the IS (**46**) and LM (**47**) equations are solved for $\boldsymbol{Y}$ and $\boldsymbol{R}$ in the Keynesian one-good version of Hicks' two-good model it is only necessary to solve one equation (**51**) to obtain $\boldsymbol{N}$. Once $\boldsymbol{N}$ is determined $\boldsymbol{P}$ is implied by (**50**), $\boldsymbol{Y^i}$ (i.e., the nominal value of investment, $\boldsymbol{PI}$) by (**47**), and, $\boldsymbol{Q}$ by (**49**), all in terms of the exogenously determined money wage $\boldsymbol{W}$ and stock of money $\boldsymbol{M}$**.** (cf., Klein, 1966, pp. 59, 63, 73-5, 193-4; Patinkin, 1965; Ackley)

5.3 Deriving Hicks' Model from Keynes' Model

Setting aside units of measurement (one can multiply through by the wage-unit $\boldsymbol{W_t}$ where appropriate if one wishes), it is easily demonstrated that Hicks' two-good model as outlined above (hence, the Keynesian one-good model as well) can, in principle, be derived from Keynes' model outline in **Table 1** above by setting Keynes savings function (**12**) equal to Keynes' MEC function (**16**) to obtain Hicks' IS savings/investment equilibrium condition:

$$\boldsymbol{s(Y_t^w) = i(R_t, P_t^a)} \qquad (\mathbf{52})$$

Hicks' LM monetary equilibrium condition is implied by the assumption that when the system is in equilibrium the quantity of money demanded $\boldsymbol{M_t^{wd}}$ (**2**) in Keynes' model must be equal to the exogenously determined stock of money in existence $\boldsymbol{M_t^w}$ (**3**):

$$\boldsymbol{M_t^w = m^d(Y_t^w, R_t)}. \qquad (\mathbf{53})$$

Given an exogenously determined money wage $\boldsymbol{W_t}$, stock of money $\boldsymbol{M_t^w}$, and the price $\boldsymbol{P_t^a}$ and stock $\boldsymbol{A_t^w}$ of non-debt assets (**52**) and (**53**) can be solved for the equilibrium values of $\boldsymbol{Y_t^w}$ and $\boldsymbol{R_t}$ which—given the assumption that effective demand adjusts to actual demand—can be substituted into (**11**) and (**16**) to obtain the equilibrium output of consumption $\boldsymbol{C_t^w}$ ($= \boldsymbol{C_t^{we}}$=$\boldsymbol{C_t^{wd}}$) and investment $\boldsymbol{I_t^w}$ ($= \boldsymbol{I_t^{we}}$=$\boldsymbol{I_t^{wd}}$) goods. These values can, in turn, be substituted into (**18**) and (**19**) to obtain employment in the investment $\boldsymbol{N_t^{wi}}$ (=$\boldsymbol{N_t^{wid}}$) and consumption $\boldsymbol{N_t^{wc}}$ (=$\boldsymbol{N_t^{wid}}$) goods industries. Given $\boldsymbol{C_t^w}$, $\boldsymbol{I_t^w}$, $\boldsymbol{Y_t^w}$, and $\boldsymbol{R_t}$ the equilibrium prices of investment $\boldsymbol{P_t^i}$ and consumption $\boldsymbol{P_t^c}$ goods are implied by Keynes' market equilibrium functions (**15**) and (**10**). These equilibrium values can be solved for in terms of the exogenously determined money wage $\boldsymbol{W_t}$, stock of money $\boldsymbol{M_t^w}$, and the price $\boldsymbol{P_t^a}$ and stock $\boldsymbol{A_t^w}$ of non-debt assets given the assumption that the effective demands $\boldsymbol{C_t^{we}}$ and $\boldsymbol{I_t^{we}}$ are equal to their actual demands $\boldsymbol{C_t^{wd}}$ and $\boldsymbol{I_t^{wd}}$ and that these demands are equal to the actual outputs of consumption $\boldsymbol{C_t^w}$ and investment $\boldsymbol{I_t^w}$ goods produced.

As with Hicks' model, the one-good Keynesian models of neoclassical economics are, in general, also implicit in Keynes aggregate model outlined above. They are, in effect, special *static* cases of Keynes' general model, but this is where the similarity between Keynesian/neoclassical economics and the economics of Keynes ends. The difference can be seen by comparing the way in which a flexible money wage is assumed to affect the level of employment in the Keynesian version of Hicks' model as modified by (**49**) - (**51**) above with the way in which a flexible money wage is assumed to affect the level of employment in Keynes' general theory as outlined above.

5.4 Changes in the Money Wage in NeoClassical/Keynesian Models

As was noted above, in the Keynesian version of Hicks' model the IS (**52**) and LM (**53**) curves can be solved for $\boldsymbol{Y}$ and $\boldsymbol{R}$, and once these values are obtained it is only necessary to solve (**51**) to obtain the level of employment $\boldsymbol{N}$ given the exogenously determined money wage $\boldsymbol{W}$. Since changes in the money wage $\boldsymbol{W}$ cannot affect either the IS (**52**) or the LM (**53**) curves in this model, by virtue of a) the assumption that the price of output $\boldsymbol{P}$ is equal to marginal cost (**50**) and b) the law of diminishing returns (**49**), (**50**) implies that there is an inverse relationship between the money wage $\boldsymbol{W}$ and employment $\boldsymbol{N}$. Thus, a fall in the money wage $\boldsymbol{W}$ must lead to an increase in employment $\boldsymbol{N}$, and, by simple arithmetic, it is possible to show that an unemployment problem can be solved by a fall in the money wage. This is taken to indicate (even to prove for some economists) that the cause of unemployment is a lack of flexibility in wages. The problem is, there is no way to explain how or why the increase in employment that is supposed to result from a fall in the money wage can come into being within the Keynesian/neoclassical paradigm since *there is no way to explain how the system gets from one point of equilibrium to another other than through the invocation of a mythical Walrasian auctioneer*.

Keynes explained the fallacy involved in the classical explanation of the effects of a change in the money wage on employment as follows:

> In any given industry we have a demand schedule for the product relating the quantities which can be sold to the prices asked; we have a series of supply schedules relating the prices which will be asked for the sale of different quantities on various bases of cost; and these schedules between them lead up to … the demand schedule for labour in the industry relating the quantity of employment to different levels of wages.… This

conception is then transferred without substantial modification to industry as a whole....

> If this is the groundwork of the argument (and, if it is not, I do not know what the groundwork is), surely it is fallacious. *For the demand schedules for particular industries can only be constructed on some fixed assumption…as to the amount of the aggregate effective demand…. [But] the precise question at issue is whether the reduction in money-wages will or will not be accompanied by the same aggregate effective demand as before* [*emphasis* added]…. [I]f the classical theory is not allowed to extend by analogy its conclusions in respect of a particular industry to industry as a whole, it is wholly unable to answer the question what effect on employment a reduction in money-wages will have. *For it has no method of analysis wherewith to tackle the problem* [*emphasis* added]…. (1936, pp. 258-60)

Keynes' point here is that the classical theory simply *assumes* that a decrease in the money wage will lead to an increase in the effective demand for output in all industries *that will be accompanied by an increase in the actual demand for output in all industrie*s without any analysis as to how the decrease in the money wage will affect incomes, prospective yields on investment, the solvency of debtors, or countless other ways in which a decrease in the money wage can affect the actual demands for goods. *There is no way to explain how or why the actual demands for goods will increase in response to a fall in the money wage within the classical theory even if there is an increase in the effective demands in this situation.*

Keynes' arguments in this passage apply with equal force to the Keynesian/neoclassical analysis of this problem. There is no way to explain how a fall in the money wage will lead to an increase in employment in Keynesian/neoclassical economics except by ignoring the effects of a fall in the money wage on demand and *just assuming* that wages and prices adjust automatically in such a way that employment increases along a falling marginal product of labor function (**50**)—that is, without assuming a mythical auctioneer is adjusting prices and quantities in such a way as to allow the system to move *instantaneously* to its short-run equilibrium values *at each point in time* as the money wage and prices fall *through time*. In other words, it is impossible to provide a logically consistent, *causal* explanation as to how the new equilibrium is achieved within the context of Keynesian/neoclassical economics; the problem is simply assumed away through the invocation of a mythical auctioneer.

6. Concluding Observation

Keynes did not view the economic system as being determined by a set of simultaneous equations but by those decision-making units that actually have the power to affect the system at each point in time. This way of looking at the economy made it possible for him to establish *the temporal order in which events must occur* and, thereby, to undertake a *causal* analysis of the *dynamic* behavior by way of "an organized and orderly method of thinking out particular problems, … isolating the complicating factors one by one" and after reaching provisional conclusions going back, as well as he could, to account "for the probable interactions of the factors amongst themselves" in an attempt to understand "the complexities and interdependencies of the real world." (Keynes 1936, pp. 297-8)

This was Keynes' method of analysis throughout *The General Theory of Employment, Interest, and Money* as he followed the example set by Marshall. It is the inability or unwillingness of neoclassical economists to examine economic problems in this way that led to

their downfall as they advocated the policies that led to the economic, political, and social problems we face today—problems that were the inevitable result of economic policies that ignored Keynes' analysis in *The General Theory* and led directly to the Crash of 2008 and the economic stagnation that followed. (Keynes 1936; Blackford 2021; 2022)

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Appendix: List of Equations

Behavioral Equations

$$Y_t^w = \frac{P_t^c C_t}{W_t} + \frac{P_t^i I_t}{W_t} \quad (1)$$
$$= C_t^w + I_t^w$$
$$M_t^{wd} = m^d(Y_t^w, R_t), \quad m_1^d > 0, \ m_2^d < 0 \quad (2)$$
$$M_t^{ws} = m^s(R_t), \quad m^{s'} > 0 \quad (3)$$
$$A_t^{ws} = \frac{P_t^a A_t}{W_t} \quad (4)$$
$$A_t^{wd} = a^d(P_t^a, R_t, Y_t^w), \quad a_1^d, a_2^d < 0, \ a_3^d > 0 \quad (5)$$
$$A_t^w = a^d(P_t^a, R_t, Y_t^w) \quad (6)$$
$$P_t^a = a(A_t^w, R_t, Y_t^w), \quad a_1, a_2 < 0, \ a_3 > 0 \quad (7)$$
$$P_t^{cs} = c^{sp}(C_t^w), \quad c^{sp'} > 0 \quad (8)$$
$$P_t^{cd} = c^{dp}(C_t^w, Y_t^w), \ c_1^{dp} < 0, \ c_2^{dp} > 0 \quad (9)$$
$$c^{sp}(C_t^w) = c^{dp}(C_t^w, Y_t^w) = P_t^c \quad (10)$$
$$C_t^{wd} = c(Y_t^w), \quad 0 < c' < 1 \quad (11)$$
$$S_t^w = Y_t^w - c(Y_t^w) \quad (12)$$
$$= s(Y_t^w), \quad 0 < s' < 1.$$
$$P_t^{is} = i^{sp}(I_t^w), \quad i^{sp'} > 0 \quad (13)$$
$$P_t^{id} = i^{dp}\left(I_t^w, R_t, P_t^a, c(Y_t^w)\right) \quad (14)$$
$$= i^{dp}(I_t^w, R_t, P_t^a, Y_t^w), \quad i_1^{dp}, \ i_2^{dp} < 0, \ i_3^{dp} \ i_4^{dp} > 0.$$
$$i^{sp}(I_t^w) = i^{dp}(I_t^w, R_t, P_t^a, Y_t^w) = P_t^i \quad (15)$$
$$I_t^{wd} = i(R_t, P_t^a, Y_t^w), \quad i_1 < 0, \quad i_2, i_3 > 0 \quad (16)$$
$$Y_t^{wd} = C_t^{wd} + I_t^{wd} \quad (17)$$
$$= c(Y_t^w) + i(R_t, P_t^a, Y_t^w).$$
$$N_t^{wid} = n^{id}(I_t^{we}), \quad n^{id'} > 0 \quad (18)$$
$$N_t^{wcd} = n^{cd}(C_t^{we}), \quad n^{cd'} > 0 \quad (19)$$
$$N_t^w = n^{id}(Y_t^{we}) + n^{cd}(Y_t^{we}) \quad (20)$$
$$= n(Y_t^{we}), \quad n' = 1.$$
$$Y_t^{we} = C_t^{we} + I_t^{we} \quad (21)$$
$$Y_t^{ws} = n^{-1}(N_t^w), \quad n^{-1'} = 1 \quad (22)$$

$$Y_t^{wd} = c\left(n^{-1}(N_t^w)\right) + i\left(R_t, P_t^a, n^{-1}(N_t^w)\right) \quad (23)$$
$$= y^d(N_t^w, R_t, P_t^a), \quad y_1^d, y_3^d > 0,\ y_2^d < 0.$$

Dynamic Adjustment Functions

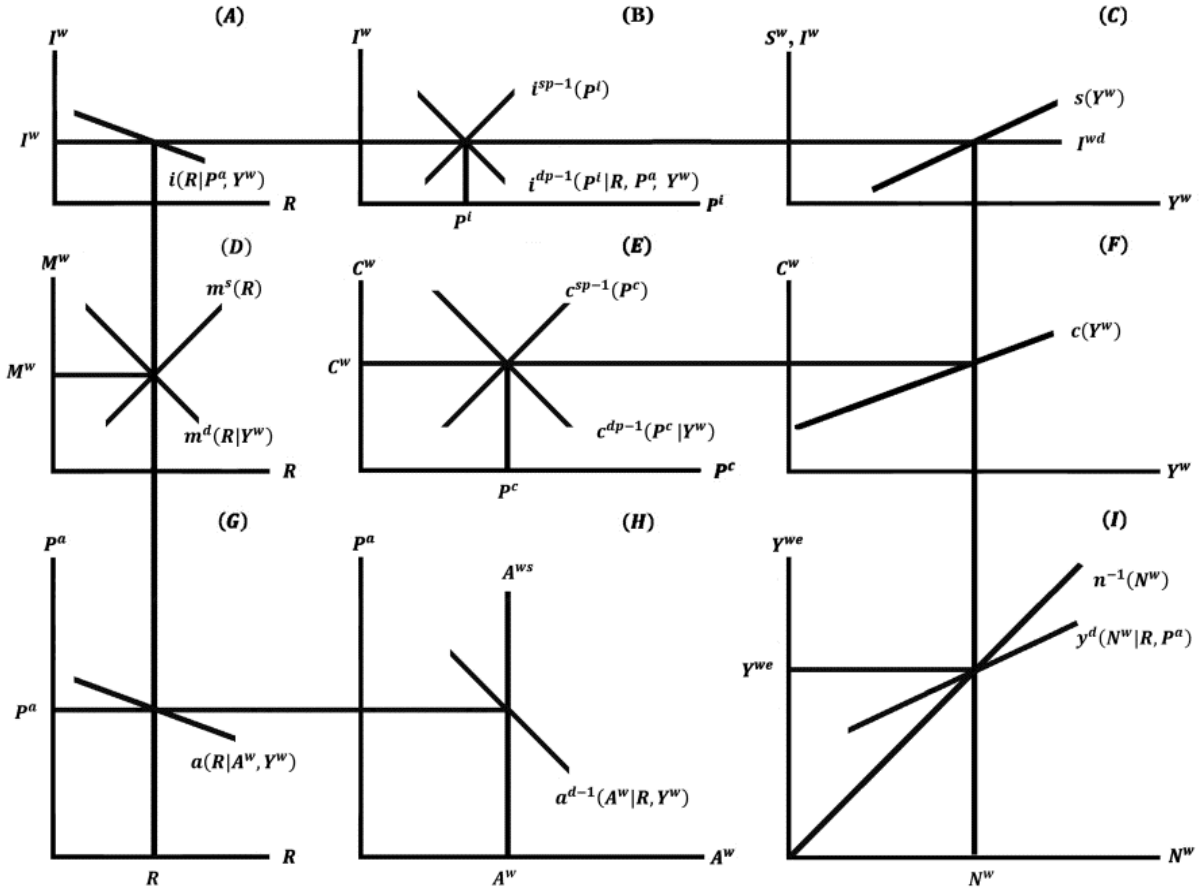
$$dR_t = g^r\left(M_t^{wd} - M_t^{ws}\right) \quad (24)$$
$$= g^r\left(m^d(Y_t^w, R_t) - m^s(R_t)\right).$$
$$dM_t = g^m\left(M_t^{wd} - M_t^w\right) \quad (25)$$
$$= g^m\left(m^d(Y_t^w, R_t) - M_t^w\right),$$
$$dP_t^a = g^{pa}\left(A_t^{wd} - A_t^{sw}\right) \quad (26)$$
$$= g^{pa}\left(a^d(P_t^a, R_t, Y_t^w) - A_t^w\right).$$
$$dC_t^{we} = g^{ce}\left(C_t^{wd} - C_t^{we}\right) \quad (27)$$
$$= g^{ce}\left(c^{dp-1}(P_t^c, Y_t^w) - C_t^{we}\right)$$
$$dI_t^{we} = g^{ie}\left(I_t^{wd} - I_t^{we}\right) \quad (28)$$
$$= g^{ie}\left(i^{dp-1}\left(P_t^i, R_t, P_t^a, Y_t^w\right) - I_t^{we}\right)$$
$$dC_t^w = g^c(C_t^{we} - C_t^w) \quad (29)$$
$$dI_t^w = g^i(I_t^{we} - I_t^w) \quad (30)$$
$$dP_t^i = g^{pi}\left(I_t^d - I_t^s\right) \quad (31)$$
$$= g^{pi}\left(i^{dp-1}\left(P_t^i, R_t, P_t^a, Y_t^w\right) - i^{sp-1}\left(P_t^i\right)\right)$$
$$dP_t^c = g^{pc}\left(C_t^d - C_t^s\right) \quad (32)$$
$$= g^{pc}\left(c^{dp-1}(P_t^c, Y_t^w) - c^{sp-1}(P_t^c)\right).$$
$$dN_t^w = g^{nd}(Y_t^{we} - Y_t^{ws}) \quad (33)$$
$$= g^{nd}\left(Y_t^{we} - n^{-1}(N_t^w)\right)$$
$$dY_t^{we} = dC_t^{we} + dI_t^{we} \quad (34)$$
$$= g^{ce}\left(c^{dp-1}\left(P_t^{cd}, Y_t^w\right) - C_t^{we}\right) + g^{ie}\left(i^{dp-1}\left(P_t^{id}, R_t, P_t^a, Y_t^w\right) - I_t^{we}\right)$$
$$dY_t^w = dC_t^w + dI_t^w \quad (35)$$
$$= g^c(C_t^{we} - C_t^w) + g^i(I_t^{we} - I_t^w).$$

Structure of Keynes' Aggregate Model

Table 1: Structure of Keynes' Aggregate Model			
Market	**Equilibrium Conditions**		**Endogenous Variables**
Assets	$M_t^{ws}= M_t^w$	$M_t^{ws}= M_t^{wd}$	M_t^w, R_t, P_t^a
	$A_t^{wd}=A_t^w$		
Investment	$I_t^{we}= I_t^{wd}$	$I_t^{we}=I_t^w$	I_t^{we}, I_t^w, P_t^i
	$P_t^{id} = P_t^{is}$		
Consumption	$C_t^{we}= C_t^{wd}$	$C_t^{we}=C_t^w$	C_t^{we}, C_t^w, P_t^c
	$P_t^{cd} = P_t^{cs}$		
Labor	$Y_t^{ws}=Y_t^{we}$		N_t^w
Identities	$Y_t^{we} = C_t^{we} + I_t^{we}$		Y_t^{we}, Y_t^w
	$Y_t^w = C_t^w + I_t^w$		

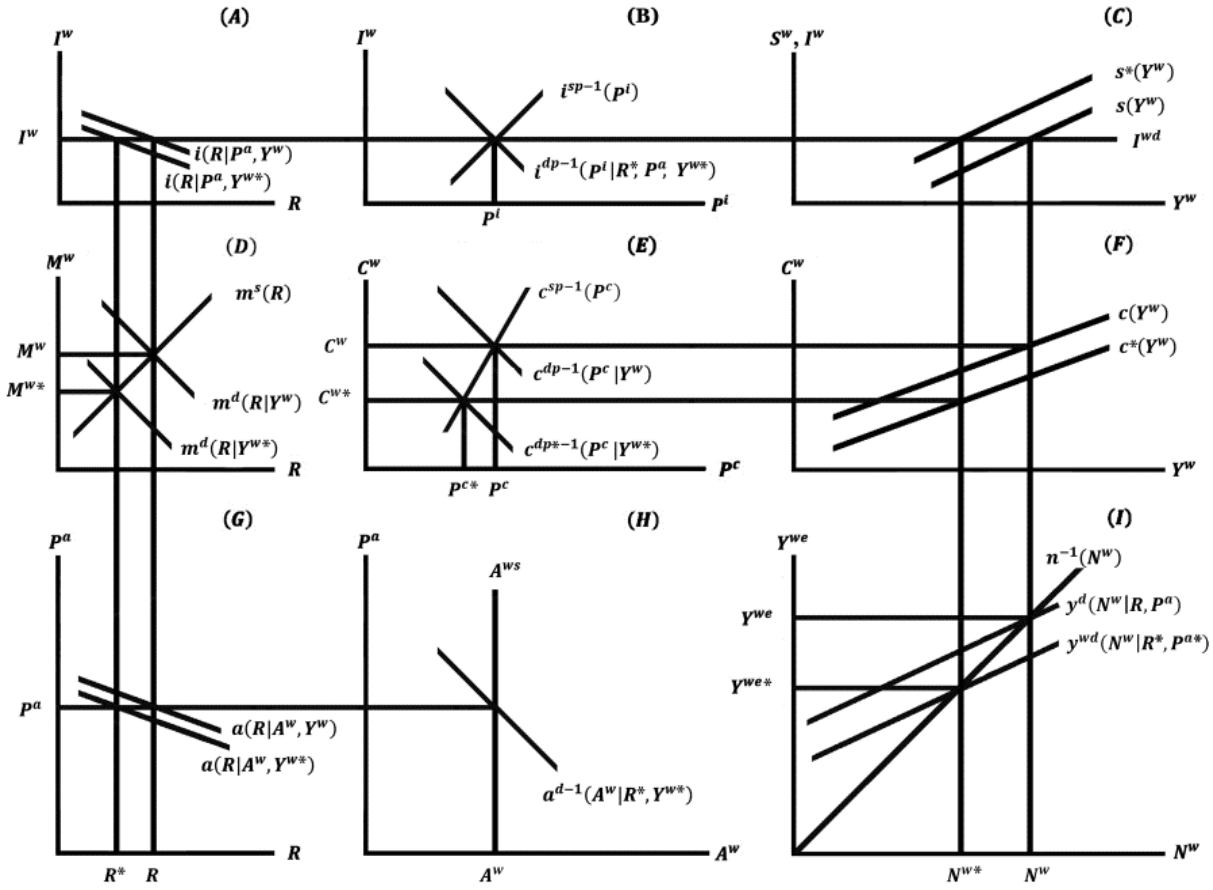
Achieving Equilibrium

Figure 1: Short-Run Equilibrium.



Changes in Short-Run Equilibrium

Figure 2: An Increase in Thriftiness.



Achieving Long-Run Equilibrium

$$P_t^{id} = i^{dp}(I_t^w, R_t, P_t^a, Y_t^w, A_t^w), \quad i_3^{dp},\ i_4^{dp} > 0, \quad i_1^{dp},\ i_2^{dp},\ i_5^{dp} < 0 \qquad (14a)$$

$$I_t^{wd} = i(R_t, P_t^a, Y_t^w, A_t^w), \quad i_2, i_3 > 0, \quad i_1, i_4 < 0 \qquad (16a)$$

$$Y_t^{wd} = c\left(n^{-1}(N_t^w)\right) + i(R_t, P_t^a, n^{-1}(N_t^w), A_t^w) \qquad (23a)$$

$$= y^d(N_t^w, R_t, P_t^a, A_t^w), \quad y_1^d, y_3^d > 0, \quad y_2^d, y_4^d < 0$$

$$P_t^{cs} = P^{c*} \qquad (8a)$$

$$P_t^{is} = P^{i*} \qquad (13a)$$

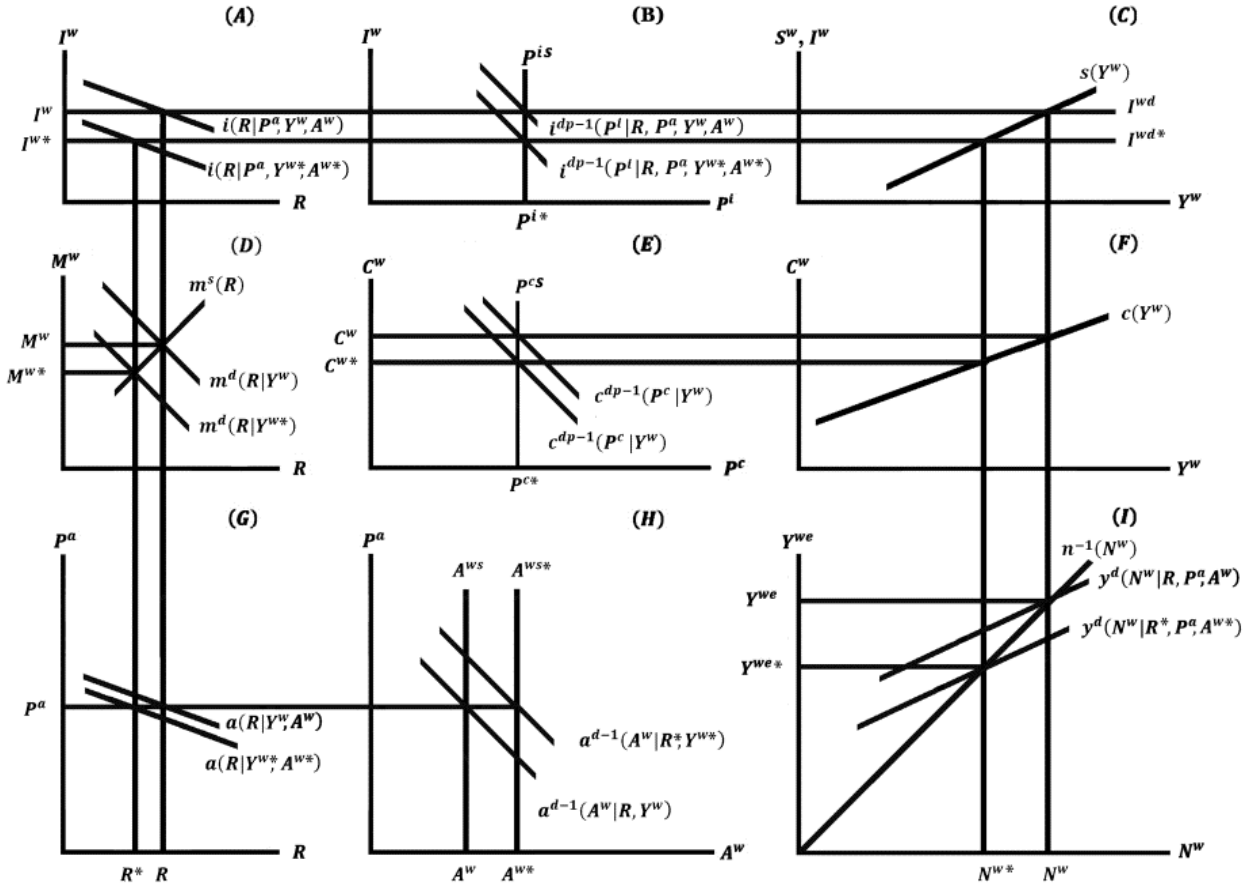
$$dI_t^{we} = g^{ie}\left(I_t^{wd} - I_t^{we}\right) \qquad (28a)$$

$$= g^{ie}\left(i^{dp-1}\left(P_t^{i*}, R_t, P_t^a, Y_t^w, A_t^w\right) - I_t^{we}\right)$$

$$dY_t^{we} = dC_t^{we} + dI_t^{we} \qquad (34a)$$

$$= g^{ce}\left(c^{dp-1}(P_t^c, Y_t^w) - C_t^{we}\right) + g^{ie}\left(i^{dp-1}\left(P_t^i, R_t, P_t^a, Y_t^w, A_t^w\right) - I_t^{we}\right).$$

Figure 3: Toward Long-Run Equilibrium.



Hicks' Two-Good Model

$$C = f^c(N^c), \qquad f^{c'} > 0, \quad f^{c''} < 0 \tag{36}$$

$$I = f^i(N^i), \qquad f^{i'} > 0, \quad f^{i''} < 0 \tag{37}$$

$$P^c = W/f^{c'}(N^c), \qquad \partial P^c/\partial N^c > 0 \tag{38}$$

$$P^i = W/f^{i'}(N^i), \qquad \partial P^i/\partial N^i > 0 \tag{39}$$

$$Y^c = P^c f^c(N^c) \tag{40}$$
$$= W f^c(N^c)/f^{c'}(N^c), \qquad \partial Y^c/\partial N^c > 0$$

$$Y^i = P^i f^i(N^i) \tag{41}$$
$$= W f^i(N^i)/f^{i'}(N^i), \qquad \partial Y^i/\partial N^i > 0$$

$$Y = P^c f^c(N^c) + P^i f^i(N^i) \tag{42}$$
$$= W f^c(N^c)/f^{c'}(N^c) + W f^i(N^i)/f^{i'}(N^i).$$

$$M = kY \tag{43}$$

$$Y^i = i(R), \qquad i' < 0 \tag{44}$$

$$Y^i = s(Y, R), \qquad s_1 > 0, \quad s_2 < 0 \tag{45}$$

$$M = m^d(Y, R), \quad m_1^d > 0, \; m_2^d < 0 \tag{46}$$

$$Y^i = i^*(Y, R), \qquad i_1^* > 0, \; i_2^* < 0 \tag{47}$$

$$i^*(Y, R) = s(Y, R) \tag{48}$$

Keynesian One-Good Model

$$Q = f(N), \qquad f' > 0, \; f'' < 0 \tag{49}$$

$$Y = Pf(N) \tag{51}$$

$$= Wf(N)/f'(N), \quad \partial Y/\partial N > 0$$

Deriving Hicks' Model from Keynes' Model

$$s(Y_t^w) = i(R_t, P_t^a) \tag{52}$$

$$M_t^w = m^d(Y_t^w, R_t) \tag{53}$$